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VEDIC MATHEMATICS

LECTURE NOTES

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कृपया पुस्तक के ऊपर कोई निशान आदि
न लगायें।

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पुस्तक—वितरण की तिथि नीचे अंकित है। इस तिथि सहित २० वें दिन तक यह पुस्तक पुस्तकालय में वापिस आ जानी चाहिए। अन्यथा १० पैसे के हिसाब से विलम्ब–दण्ड लगेगा।

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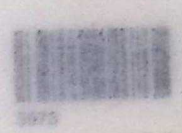
International Course on Elementary VEDIC MATHEMATICS



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-: VEDIC MATHEMATICS SUTRAS :-

1. By one more than the previous one
2. All from nine & last from ten
3. Vertically & Crosswise
4. Transpose & Apply
5. The summation is equal to zero
6. If one is in ratio, the other one is zero
7. By addition and subtraction
8. By the completion and noncompletion
9. Sequential motion
10. The deficiency
11. Whole as one & one as whole
12. Remainder by the last digit
13. Ultimate and twice the penultimate
14. By one less than the previous one
15. The whole product is the same
16. Collectivity of multipliers

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ANCIENT VEDIC MATHEMATICS

SALIENT FEATURES

16 Vedic sutras of 120 words can solve all mathematical problems.

With a firm scientific footing

A revolution in concept of learning

Opening new areas of R & D

The Vedic Maths can be used to check the Computer results, and frequently supersedes calculators

Revealing hither-to unknown, Great and Mysterious Qualities of Numbers

Simple, short mental mathematical processes often with magic speed

Requires no prerequisite courses

Mathematics otherwise burden, turns to be a joyful subject

A boon for all competitions

More useful, accurate and applicable at times, compared to modern computers

Speed of learning maths is nearly ten times

A Complete System

A most natural Vedic system, which developed the human computer to wonderful levels.

The problem solutions flow naturally to the mind with effortless efforts.

A system with mental multichoice procedures, which keep the mind alert and agile. VM has a unique independent cross checking system.

A system which children learn with smiles and most often ask, 'why Vedic Maths is not being taught in every school and college in the whole world.

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THE SIXTEEN SUTRAS OF VEDIC MATHEMATICS

सूत्र :-

१ एकाधिकेन पूर्वेण	1. Ekadhikena Purvena (also a corollary)
२ निखिलं नवतश्चरमं दशतः	2. Nikhilam Navatascaramam Dasatah
३ ऊर्ध्वतिर्यग्भ्याम्	3. Urdhva-tiryagbhyam
४ परावर्त्य योजयेत्	4. Paravartya Yojayet
५ शून्यं साम्यसमुच्चये	5. Sunyam Samyasamuccaye
६ (आनुरुप्ये) शून्यमन्यत्	6. (Anurupye) Sunyamanyat
७ संकलनव्यवकलनाभ्याम्	7. Sankalana-vyavakalanabhyam (also a corollary)
८ पूरणपूरणाभ्याम्	8. Puranapuranabhyam
९ चलनकलनाभ्याम्	9. Calana-Kalanabhyam
१० यावदूनम्	10. Yavadunam
११ व्यष्टिसष्टिः	11. Vyastisamastih
१२ शेषाण्यङ्केन चरमेण	12. Sesanyankena Caramena
१३ सोपान्त्यद्वयमन्त्यम्	13. Sopantyadvayamantyam
१४ एकन्यूनेन पूर्वेण	14. Ekanyunena Purvena
१५ गुणितसमुच्चयः	15. Gunitasamuccayah
१६ गुणकसमुच्चयः	16. Gunakasamuccayah

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THE FOUNDATIONS OF VEDIC MATHEMATICS

Computational techniques are just one aspect of the complete approach of Vedic Mathematics. Maharishi's Vedic Science locates the deepest value of Vedic Mathematics in the mathematical structure of consciousness itself. The founders of the Vedic tradition discovered the capability of the human mind to settle into a state of deep silence while remaining awake, and therein to experience a completely unified, simple, and unbounded state of awareness, called pure consciousness. In this experience, Maharishi explains, the conscious mind becomes identified with an infinite, all-prevading unified field, an eternal continuum underlying all existence.

Maharishi describes the Ved, within the field of pure consciousness, as existing on the level of undivided unity. Within this unity two values, existence and intelligence, can be located. It is intelligence value that knows itself and thereby conceptually creates three values within itself-the knower, the process of knowing, and the known. Thus, one becomes two, and two becomes three. Through the further interaction of these three values, the infinite diversity of the universe is created from the sequential quantification of unity. This unfoldment is portrayed with perfect precision by Vedic Mathematics.

According to Vedic Science, all values of relationship and transformation, both abstract and concrete, are simply expression of the self-interacting structure and dynamics of an underlying, totally unified field of pure intelligence. The unlimited variety of relationships and transformations, ranging from the most abstract areas of modern mathematics to the most concrete phenomena of nature, can thereby be described in terms of the inner relationships of pure consciousness. Vedic Mathematics provides this description.

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VEDIC MATHEMATICS
THE COSMIC SOFTWARE FOR THE COSMIC COMPUTER

INTRODUCTION

Vedic Mathematics offers a new holistic approach to mathematics and to mathematics education. Its range extends from the most concrete values of numerical computation to the most abstract aspects of the dynamics of intelligence.

The computational methods of Vedic Mathematics have been found to be easier to learn, faster, and more enjoyable than conventional methods. The intrinsic flexibility of the Vedic system of calculation develops the students' creativity and clearity of mind.

The system of Vedic Mathematics is derived from the Vedic tradition of knowledge. At its deepest level, Vedic Mathematics is the exploration and exposition of the self-interacting structure of pure consciousness, which is the orderly basis of all phenomena in nature. By connecting mathematics to its source in the field of pure consciousness, Vedic Mathematics has the potential to bring fulfillment to all areas of mathematics and to solve the current crisis in mathematics education.

THE SUTRAS OF VEDIC MATHEMATICS

All the computational methods of Vedic Mathematics are based on sixteen sutras (aphorisms) that are directly derived from the Atharv-Ved, one of the primary texts of the ancient Vedic literature of India. This mental computational system was brought to light in recent times by the Shankaracharya of Puri, Swami Bharati Krishna Tirtha.

On one level, each sutra aphoristically describes the actual computational method to be used for the various types of mathematical problems. On a deeper level, however, each sutra may be seen as a refined formula for producing a high degree of coherence and order in brain functioning, facilitating the rapid and precise solution to the mathe - matical problem.

The coherently functioning brain, with its billions of neurons and interconnections is, in Maharishi's words, a "cosmic computer". Through proper programming it can accomplish anything. The sutras of Vedic Mathematics are the "cosmic software" that create the ability to compute most rapidly and precisely.

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APPLICATIONS TO MATHEMATICS EDUCATION

Vedic Mathematics offers a new approach to resolving the current crisis in mathematics education. It is not simply a collection of new computational techniques; rather, it provides an entirely different approach to mathematical computation, based on pattern recognition.

The conventional methods of mathematical computation consist of rigid, sometimes monotonous procedures that are uniformly applied to all problems of a given type. The result is often boredom, strain and a lack of enjoyment for both student and teacher.

By contrast, the techniques of Vedic Mathematics allow for constant expression of a student's creativity. First, for any type of problem, there are always several different approaches : at least one general technique, application to all cases, and also a number of special techniques, applicable to cases having a particular pattern. The general technique is always faster and easier than the conventional technique, while the techniques for special cases work extraordinarily fast, yielding solutions to complex problems, often at the speed of just writing down the successive digits of the answer. Second, at each stage of a particular technique, there are usually options with regard to the next step. This flexibility in the method keeps the mind lively and alert and cultures the ability to quickly discover the path of least action on the way to the solution.

In addition, the techniques of Vedic Mathematics have been found to be easier to learn than conventional methods. Computational complexities are significantly reduced by using digits and their complements. And finally, unique, independent cross-checking methods make the system less prone to error.

In summary, then, the techniques of Vedic Mathematics not only provide a quick and accurate solution to the problem at hand, but also develop clarity of mind and intuition In this way, Vedic Mathematics succeeds in fulfilling the two major goals of mathematics education : to teach students procedures for solving specific mathematical problems and to develop clear and logical thinking.

The problem of "math anxiety" which confronts mathematics educators around the world, simply does not arise. The Vedic Mathematics research group in London has successfully applied Vedic Mathematics at both primary and secondary school levels. The Indian Ministry of Human Resource Development recently held a workshop on Vedic Mathematics at Rajasthan University, which was attended by mathematicians, educators, and government representative. The workshop recommnended that the government

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should set up a national committee to followup further work in Vedic Mathematics and plan its implementation in the teaching in schools and colleges. Further, the government funding agencies like University Grants Commission etc. should be asked to provide funds for further research and development of Vedic Mathematics. The national committee is already started organising series of workshops on Vedic Mathematics. These workshops are being organised by the Rashtriya Ved Vidya Pratishthan.

The general principle is familiar to modern mathematicians. The unfied foundation of modern mathematics is provided by set theory, the mathematics of the infinite. At the foundation of set theory are mathematical principles that directly describe the nature of an abstract, infinitely dynamic field of intelligence. All of mathematics is described as sequentially unfolding from the internal dynamics of this field of pure intelligence. For example, all sets are sequentially created from the unmanifest point value of the null set. This self-referral nature of the field of intelligence at the basis of set theory is expressed at a deeper level by the reflection principle, which underlies the mathematical description of the different quantified values of infinity provided by the theory of large cardinals.

In Vedic Mathematics, all the fundamental principles are derived from direct experience of the unmanifest source of orderlines within the unifield field of pure consciousness. Vedic Mathematics is, therefore, the mathematics of the most fundamental structures of natural law. This is its unique stature and contribution to the field of knowledge.

INVITATION

The computational applications of Vedic Mathematics are only the first step in an ongoing research program. All areas of modern mathematics, both pure and applied, fall within the scope of Vedic Mathematics. We are just beginning to tap the enormous potential of this more fundamental and complete approach to knowledge. All are invited to join the research programme.

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VEDIC MATHEMATICS SYSTEM

Whatever we see in nature has a perfect mathematical structure. The rotation of the earth, movement of the stars and planets, growth of the plants and trees etc. are all perfectly mathematically structured. The blooming of a flower, the growth of a plant, the rotation of the earth, all go on so very effortlessly. Recent research is bringing to light the perfect mathematical structure of the Vedic texts. As a matter of fact whatever we do in life has mathematics in some form or the other. It is not possible to prepare even a delicious cup of coffee without a precise sense and application of mathematical proportion of different ingredients used. It's a pity that such a wonderful subject of fundamental and vital importance 'mathematics' is creating the problem of 'MATHS ANXIETY' in all parts of the world. The common students are afraid of Mathematics. The gravity of the problem is quite clear from a recent survey report published in America. The top 10% of the students of Maths were asked the question, "Do you like Mathematics", And 60% of them said No. The answer of the remaining 90% is quite obvious.

Further, whatever we want to do, we always have a large number of alternatives and choices in doing the same. How strange that in Mathematics there seems to be almost no choice, and, most of the time we have to follow a unique, fixed, monotonous, often boring and tedious procedure in whatever arithmetical operation we want to do. Interestingly enough this unique approach is considered to be a quality by some mathematicians. Anything fixed, naturally becomes monotonous and boring and Maths Anxiety is the natural result. Vedic Maths on the other hand with its multiple choice makes maths a playful and interesting subject, which naturally brings frequent smiles on the face and joy in the heart of the students.

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The sixteen Vedic sutras provide a structured system which is applicable in, and, hence, is capable of explaining almost any facet of cosmos, both at abstract as well as applied level. These sutras in their philosophical interpretation even explain the functioning of human psychology. In their simple applied form, their crude translation in English has been presented in the beginning.

Sanskrit is a rich language and for deeper meaning one has to dive deep into the original sutras. The first Vedic Sutra Ekadhikena Purvena in simple English means by one more than the previous one, and a couple of very interesting and wonderful applications are presented in Pushp I. However the same sutra EkadhikenaPurvena(एकाधिकेन पूर्वेण) explains the whole concept of growth and development of an individual, society or any facet of knowledge. The word purvenaessentially means the previous state. Ekadhiken means one higher than (the previous state). The higher state again becomes the previous and we go higher and higher and higher step by step, and that speaks out the whole process of growth and development. Similarly each sutra has a deep philosophical meaning. However, in the initial Pushps we are only looking at the simple applied facets of these sutras.

Maths with Smiles

Vedas are the source of all knowledge and of all Sciences. They are not only the oldest but also the best. The Vedas have guided millions of aspirants on the path of knowledge. With the passage of time most of the Vedic knowledge has been scattered and shattered. Vedic Mathematics is no exception to it. It is inspiring to observe that, deep interest is being taken, the world over in reviving the Vedic Sciences.

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According to the brain specialists the left half of the brain systematically collects the information, does sequential analysis and arrives at the logical conclusions and results. Only left half of the brain is activated and developed by the present Maths education, being given in the schools and colleges. The right half of the human brain works with pattern recognition and has the capability of intuition. This most powerful facet of human personality the intuitive faculty, unfortunately, remains undeveloped in most of the students. In Vedic Mathematics system, the very first step is to recognize the pattern of the problem and pick up the most efficient Vedic algorithm. Further, at each subsequent step, we recognize the pattern and complete the task by using the appropriate superfast, mental working procedure. As in Vedic Maths we have multiple choice available at each stage of working. This practice, systematically develops the right half of the brain as well. As such by the regular practice of the most natural mental procedures of Vedic Maths system, the **HO-LISTIC DEVELOPMENT OF THE HUMAN BRAIN** automatically takes place. In a nutshell we can say that the Vedic Maths provides the cosmic software for the cosmic computer (the human mind), so that the later is not wasted.

IN A NUT SHELL

Vedic Maths (VM) offers a new and entirely different approach to mathematics based on **pattern recognition** for resolving the current crisis in Maths education. VM allows for constant expression of a **student's creativity** and has been found to be **easier to learn.** For any problem there is always one general technique applicable to all cases. There are also number of special techniques for special pattern problems. The element of choice and flexibility at each stage keeps the mind lively and alert and develops **clarity** of **mind** and **inuition**, and, thereby Holistic development of the human brain automatically takes place.

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SWAMI BHARATI KRISHNA TIRTHA

Swami Bharati Krishna Tirtha, author of the first recent Vedic Mathematics book was the Shankaracharya of Goverdhan Peath, Jagadguruji, named Venkatraman in his early days, was an exceptionally brilliant student and invariably won the first place in all the subjects in all the classes throughout his educational carrier.

In 1904, at the age of just twenty, he passed M.A., examination in seven subjects simultaneously securing the highest honours in all, which is perhaps the all time world-record of academic brilliance. His subjects included Sanskrit, Philosophy, English, Mathematics, History and Science.

In the words of Swami Ji, "These 16 Sutras (aphorisms) of barely 120 alphabets, apply to and cover each and every chapter of each and every branch of mathematics (including arithmetic, algebra, geometry - plane and solid, trigonometry - plane and spherical, conics - geometrical and analytical, astronomy, calculus etc.,). In fact, there is no part of mathematics, pure or applied, which is beyond their jurisdiction The Sutras are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarised in one word "mental" .

Vedic Mathematics has originated from that Ved.Ved is all Divine knowledge. Any knowledge derived from the Ved is bound to have the touch of the divine bliss. Therefore the very natural, easy and superfast algorithms of Vedic Mathematics bring an upsurge of joy and bliss. Due to its very special and universal features, Vedic Maths system converts the dry and tedious Maths into a playful and joyful subject, which children learn with smiles. Therefore, the Vedic Maths is the gift of the Ved to solve the problem of "MATHS ANXIETY" being faced in maths education, in the whole world.

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WORK AT ROORKEE

I wish to draw your kind attention to another facet of the humble research effort at the university campus. Everybody who has known a little about the Vedic Mathematics (VM) has derived national pride and shown a great zeal to learn more. VM kindles in our hearts, the Jyoti of 'shradha' towards Mother India.

The Spiritual Study Group of Roorkee University has been spreading this wonderful mathematical facet of ourrich, vast and scientific cultural heritage. On repeated requests since May 85, SSG, has already organised the VM courses for 41 batches attended by 4350 participants, ranging from school children, teachers, university students, research scholars, field engineers to the university professors, of USA Canada, Mexico, Holland, France, England, Germany, Switzerland, Spain etc. by Aug. 1988.

Since Haridwar Maha Kumbh, 1986, SSG has been conducting VM Correspondence Courses in Hindi & English for home study and new batches start from Dushera (October) and Baishaki (April) every year.

Thousands of Vedic Mathematics demonstration lectures are being delivered in different parts of the world. In India, hundreds of lectures have already been given in Delhi, Punjab, Gujrat, West Bengal, U.P., M.P., Kerala, A.P., Goa, Rajasthan, Hariyana and Maharashtra. Further, demonstration lectures and seminars on Vedic Mathematics have already been delivered in several cities of America, England, Canada, Mexico, Norway, Sweden, Finland, Denmark, Hungry, Germany, Italy, Switzerland, Spain, Holland in the recent past.

SSG Publications

In order to provide comprehensive details of this wonderful, ancient Bhartiya Cultural Heritage SSG is publishing a series of books in Hindi and English.

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SYMPOSIUM AND WORKSHOPS

A Symposium on Vedic Mathematics was held in Feb. 1985 at Nagpur. The Second at Bhartiya Vidya Bhawan's London Centre in Sept. 1986. The first 11-day workshop on Vedic Maths, and its computational potentialities was held at University of Roorkee in Aug. 1987. A 3-day workshop was organised in Dec. 1987 by SSG at Banaras Hindu University, Varanasi, followed by a 4-day workshop in Feb. 1988 at G.B. Pant University, Pantnagar. Further, Vedic Maths workshop was organised by the Ministry of Human Resource Development and Rashtriya Ved Vidya Pratishthan at Jaipur in March and at Ahmedabad in October 1988.

Recently, Vedic Maths seminars have also been held during April, 1988 at Imperial College, London ; Oxford University, Cambridge University, and Ideal Village, Lancashire ; Washington ; Harward University, Scientific Systems, Boston ; Annual NCTM meeting at Chicago ; Los Angles ; Stanford University, Pola Alto ; University of California, Berkeley ; Indiana ; Goldendome MI University Fairfield ; Toronto ; Mexico University, MIU Oslo, Academy at Lily Hamer, Norway ; Upsala University and Stockholm University, Sweden ; Turku University, Finland ; Copenhagen University and HC Oersted Institute, Denmark ; Hamburg University and Technical University Berlin ; Institute of Leadership Budapast, Hungry. Further during the month of May, 1988 Vedic Maths seminars were held at Main University Vienna, Austria, TM Centre Rome ; Engineering Faculty University of Naples and University of Milan, Italy ; MERU Seelsberg, Geneva University Switzerland ; Paris University France ; Barcelona, Spain, MERU Vlodrop, Amsterdam University, Holland. International two week foundation courses were organised at MERU Switzerland, Washington, Mexico and MIU, IOWA USA during June-July, 1988.

Ph.D. thesis on Vedic Mathematics just submitted by Mr. S.K. Kapoor.

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1. ADDITION

Introduction : The process of addition in Vedic Method is converted to a series of very simple additions, each number being always less than 10. The simple sutra Shudha' is deployed whenever the addition exceeds 9 . The step by step process is illustrated in the following examples : (Ref. 2 & 9)

Example 1 :

$$\begin{array}{r} 379 \\ 854 \\ 767 \\ 426 \\ \hline \\ \hline \end{array}$$

Step 1 : The addition is carried out column by column in the usual manner, moving from bottom to top.

(a) 6 + 7 gives 13 which is more than 10 so Shudha is said and a dot is put above 7. The tenth place, unity is dropped and only the number of the unit place (3) is carried forward. The unit place number (3) is added to the next number.

(b) 3 + 4 = 7 therefore no Shudhikaran is required at this stage. By adding the next number to the result we get 7 + 9 = 16. Once again Shudhikaran is required. We say Shudha and put a dot above the latest digit (9). The unit at tenth place is dropped and the number at unit place is the answer

$$\begin{array}{r} 37\dot{9} \\ 854 \\ 76\dot{7} \\ 426 \\ \hline 6 \\ \hline \end{array} \quad \uparrow$$

(c) The addition process in the upward direction has been used. However the addition can be carried in the downward direction with equal ease and accuracy.

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Step 2 : Before moving to the second column (10th place) the dots due to Shudhikaran in the first column (unit place) are counted (2 in this case). This is added to the lowest number 2 + 2 = 4. The result is added to the next number 4 + 6 = 10. Therefore, we say Shudha, put a dot above the latest digit (6). The unity at tenth place is dropped and the number at the unit place is carried forward (0). 0 + 5 = 5 is the result of addition of the next digit, which is added to the next number (7 which is also the last number in the column). Therefore 5 + 7 = 12. Once again Shudhikaran is carried out. We get a dot above the last number (7) and the left over digit at the unit place is the answer (2).

$$\begin{array}{r} \dot{3}\,\dot{7}\,\dot{9} \\ 8\,\dot{5}\,4 \\ 7\,\dot{6}\,\dot{7} \\ 4\,2\,6 \\ \hline 26 \\ \hline \end{array}$$

Step 3 : Before moving on to the hundredth place (the third column). The dots due to Shudhikaran' process in the second column are counted (which are 2 in this case).

(a) This is added to the lowest number 2 + 4 = 6, the resulting number is added to the next number 6 + 7 = 13, since the resulting number has extended 9, Shudhikaran' is required. Therefore we say Shudha and put dot above the latest number (7), drop the tenth place unity and carry forward the left over digit at the unit place (3).

(b) This is added to the next number 3 + 8 = 11. Therefore the latest number (8) also gets a dot of Shudhikaran and the left over number at the unit place (1) is carried forward. This when added to the subsequent number yields the answer 1 + 3 = 4.

(c) The number of dots of Shudhikaran are counted to get the carry over number at the thousandth place (2).

$$\begin{array}{r} \dot{3}\,\dot{7}\,\dot{9} \\ \dot{8}\,\dot{5}\,\dot{4} \\ \dot{7}\,\dot{6}\,\dot{7} \\ 4\,2\,6 \\ \hline 2426 \\ \hline \end{array} \quad \uparrow$$

Hence 2426 is the answer.

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2. SUBTRACTION

Introduction

The process of subtraction in Vedic Method is converted to a series of simple additions and subtractions. The sutra Shudha is deployed whenever the larger digit is to be subtracted from the smaller digit. The step by step process is illustrated in the following examples : (Ref. 2 and 9).

Example 2 :

$$\begin{array}{r} 57 \\ -\ 32 \\ \hline \\ \hline \end{array}$$

Step 1 : The digit 2 at unit place is smaller than 7.

Therefore, 7 - 2 = 5 (No Shudhikaran is required)

Step 2 : The digit 3 at tenth place is smaller than 5.

Therefore, 5 - 3 = 2 (No Shudhikaran is required).

$$\begin{array}{r} 57 \\ -\ 32 \\ \hline 25 \\ \hline \end{array}$$

Example 3 :

$$\begin{array}{r} 3752 \\ -\ 1871 \\ \hline \\ \hline \end{array}$$

Step 1 : The unit digit 1 is smaller than 2, therefore 2 - 1 gives 1 at the unit place (No Shudhikaran is required).

Step 2 : The digit at the tenth place is 7 and greater than the corresponding digit 5. Therefore we do Shudhikaran and put a dot above the next digit. Compute the difference

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of the larger number from the base 10 which is 3 in this case. Add this difference to the digit at the top to get the digit at this place. Therefore 3 + 5 = 8.

$$\begin{array}{r} 3\,7\,5\,2 \\ -\ 1\,8\,7\,1 \\ \hline 8\,1 \end{array}$$

Step 3 : Since there is a dot on next digit i.e. on 8. Therefore, this digit is increased by 1 which, in this case becomes 9. Now 9 is greater than 7 (at hundredth place). Therefore, Shudhikaran is done and put a dot above 1. Compute the difference of the larger number from the base 10 which is 10 - 9 = 1 in this case. Add this difference to the digit at the top. Therefore, 1 + 7 = 8.

$$\begin{array}{r} 3\,7\,5\,2 \\ -\ 1\,8\,7\,1 \\ \hline 8\,8\,1 \end{array}$$

Step 4 : Since there is a dot above the next digit i.e. above 1. Therefore, this digit is increased by 1, which in this case becomes 2. Now 2 is smaller than 3 and Shudhikaran is not required. The difference of 3 - 2 = 1 gives the digit at this place.

$$\begin{array}{r} 3\,7\,5\,2 \\ -\ 1\,8\,7\,1 \\ \hline 1\,8\,8\,1 \end{array}$$

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3. VEDIC CHECK FOR ADDITION AND SUBTRACTION

Introduction

In the last two chapters we have seen that even the simple operations of addition and subtraction can be done faster by using the simple Vedic sutra Shudha. One of the unique facets of the Vedic methods is the simple methods for checking the correctness of all the arithmetical operations, like addition, subtraction, multiplication and division etc. It is further interesting to note that these methods can also be extended and applied with equal ease, efficiency and accuracy even to the algebraic operations including factorization, partial fractions etc. However, at present we shall just see the step by step methods of checking the correctness of any addition or subtraction. (Ref. 2 and 9).

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Subtraction - Check

In the fifth chapter we have learnt the Vedic method of subtraction by using the sutra Shudha. We have seen that in the Vedic method there is absolutely no need to carry the digit to the lower place, as is required in the present system. The concept of carrying the digit to the lower place, we know, is very difficult to learn for the small childern.

The simple operations involved in checking the correctness of the subtraction are that we convert each of the three numbers, namely, the number, the number to be subtracted and the result, into single digit numbers. This is accomplished by adding digit by digit, the different digits of the number. As soon as this adition gives us a two-digit number, we add the two digits together to convert them into a single digit number. Further we add the equivalent single digit number of the result to the single digit number obtained by adding the number subtracted. Once again if this addition gives a two-digit number, we add the two digits together to finally get the single digit adition. The single digit number thus obtained should be

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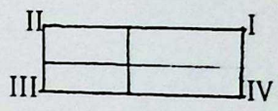
equal to the equivalent single digit number obtained from the original number.

The step by step procedure is outlined in the following examples.

Example 4 :

Say we subtract 32 from 57. We draw a square in front of the sum and divide into four squares by joining the mid-points.

$$\begin{array}{r} 57 \\ -\ 32 \\ \hline 25 \end{array}$$



Step 1 : The result 25 converted to a single-digit number gives 2+5 = 7. This is written in the lower right square (IV).

Step 2 : The number subtracted 32 similarly gives the single-digit number as 3+2 = 5 and is written in the lower left square (III).

Step 3 : Similarly, the original number 57 gives the single-digit number as 5+7 = 12, 1+2 = 3.

Since addition of 5+7 has given a two-digit result 12, we add the digits once again to finally end in the single-digit number. This is written in the upper left square (II).

3	
5	7

5 + 7 = 12,
1 + 2 = 3

Step 4 : Now we add the equivalent single-digit number of the answer (Step 1) to the single-digit number of the number subtracted (Step 2). In the present example 5+7 = 12, 1+2 = 3 therefore giving 3 as the final single-digit number. The single digit thus obtained is entered in square I. Since this is equal to the single-digit number obtained for the original number (Step 3) written in square II. We can be reasonably sure that our subtraction is correct.

3	3
5	7

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4. BASE SYSTEM

In Vedic Mathematics the base system is very frequently used to reduce the computations involved. The base is considered to be the powers of 10. For example 10, 100, 1000, 10,000 and so on are the different bases. (Ref. 2 & 9).

Deviation from the Base :-

(1) We would take, as base, for our calculations, that power of 10 which is nearest to the number.

(2) For the numbers more than the base, the portion of the number exceeding the base is written with a +ve sign in front of the number.

(3) For the number below the base, we subtract the number from the base and write down the remainder in front of the number with a -ve sign. This deviation is directly written by using the Nikhilam Sutra. (Sutra No. 2). The Sutra means All from 9 and last from 10'.

Example 5 :

Say Number is 103

The base is 100 and the number is 3 more than the base.

Therefore, 103 is written as 103 + 03

Say Number is 115

The base is 100 and the number is 15 more than the base.

Therefore, 115 is written as 115 + 15

Say Number is 1012

Now base is 1000 and the number is 12 more than the base.

Therefore, 1012 is written as 1012 + 012

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Say the Number is 92

The base is 100 and the number is 8 less than the base

Therefore 92 is written as 92 - 08

Say the Number is 991

The base is 1000 and the number is 9 less than the base

Therefore, 991 is written as 991 - 009

By Nikhilam Sutra we get 9-9 = 0, 9-9 = 0 and 10-1 = 9.

Therefore deviation = -009

Say the Number is 75

The base is 100 and the number is 25 less than the base

Therefore, 75 is written as 75 - 25

Say the Number is 971

The base is 1000 and the number is 29 less than the base

Therefore, 971 is written as 971-029

By Nikhilam Sutra we get 9-9 = 0, 9-7 = 2 and 10-1 = 9

Therefore deviation = -029

Say the Number is 9975

The base is ..... and the number is ..... less than the base

Therefore ..... is written as .....

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5. NIKHILAM SUTRA

This is the second sutra. Using this sutra we can very conveniently perform multiplication of numbers consisting of very large or small digits. In this process the Vedic Base System is used.

The simple meaning of this sutra (No. 2) is, 'All from 9 and last from 10'. This sutra is used to straight away write down the deviation of the numbers from the base for the numbers smaller than the base. Let us take an example.

Illustration 6 : Say the number is 9856. The base (i.e. the multiple of 10) close to the number 10000. The Nikhilam sutra which reads Nikhilam Navtashcaraman Dashatah tell us to substract each digit of the number from 9 and the last digit from 10.

Therefore for given number 9856 the digit by digit deviation is obtained as

9 - 9=0, 9 - 8 = 1, 9 - 5 = 4 and the last digit of the deviation is 10-6 = 4. Hence the deviation is -0144.

Similarly the deviation of 937 is -063 (Ref. 1,2 & 9).

6. VINCULUM

The vinculum number consists of both positive and negative digits.

The Approach : In the Vinculum process we take the complement of the digits bigger than 5 with a negative sign, and a positive carry over to next higher order digit.

Illustration 7 : Let us take the number 9, 9 = 10-1, therefore 9 is 1 less then 10. This we can also write as $1\bar{1}$ instead of 9. The property of the numbers is unchanged and we can add, multiply etc. in the usual manner. However, we have to keep in the mind that the $\bar{1}$ is always appearing as -1.

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Illustration 8 : 782893

$782893 = 1\bar{2}\bar{2}3\bar{1}\bar{1}3$

Normalising : Let us further consider a few examples of normalizing the Vinculum number. It shall be surprisingly interesting to learn that the Nikhilam sutra can be used even for the normalizing process. The only difference being that the previous digit is now reduced by 1.

Illustration 9 : $3\bar{3}4\bar{4}$

The complement of $\bar{3}$ is 7 and of $\bar{4}$ is 6. Further, each digit before the bar digit gets reduced by 1. So that 3-1 = 2 and 4-1 = 3.

Therefore $3\bar{3}4\bar{4} = 2736$

Illustration 10 : $3\bar{3}\bar{4}\bar{2}$

Since there are three digits in a group we can use the Nikhilam sutra to straight away write down the complement of this group, and we should remember to reduce the digit previous to this group by 1.

Therefore, by Nikhilam 9-3 = 6, 9-4 = 5 and 10-2 = 8, and 3-1 = 2.

As such $3\bar{3}\bar{4}\bar{2} = 2658$

Illustration 11 : $20\bar{3}\bar{4}0\bar{1}\bar{2}1$

$20\bar{3}\bar{4}0\bar{1}\bar{2}1 = 19659881$

Alternatively, we can normalize the above number in two steps

$20\bar{3}\bar{4}0\bar{1}\bar{2}1 = 2\bar{1}66\bar{1}881$

$= 19659881$

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Further practise shall give us the grip on this system, which is immensely useful for simplifying the computation processes.

Illustration 12 : $\bar{4}35\bar{2}\bar{7}2$

Here the digit is negative, hence it is a negative number. We may either convert all digits to bar digits, or all to + ive digits

$\bar{4}35\bar{2}\bar{7}2 = \bar{3}\bar{6}5\bar{2}\bar{6}\bar{8} = -365268$

or $\bar{1}634732 = -1000000 + 634732 = -365268$

At each stage we use Nikhilam Sutra.

Note : The convertion to Vinculum form and normal can be checked by using sutra 15. (Ref. 1, 2 & 9)

7. TABLES

The Vedic method gives us the key which works superfast not only upto tables of 20 but is equally efficiently applicable to any table even upto 1000 or more. (Ref. 1,2 & 9).

Operators

The operators of any number can be obtained by simply converting the given number into the Vinculum form, so that finally no digit is bigger than 5.

Therefore $19 = 2\bar{1}$; $2\bar{1}$ are the operators at the tenth and the unit places respectively.

Similar $89 = 1\bar{1}\bar{1}$

Therefore $\bar{1}$, $\bar{1}$, 1 are the operators at the unit place, tenth place and hundred place respectively.

The Nikhilam sutra can be used for converting any number to its Vinculum form. We can also get the Vinculum form by adding the same number to itself and than subtracting the given number from the sum without borrowing the digits from the higher place.

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19	89	82
+19	+89	+82
38	178	164
-19	-89	-82
2$\bar{1}$	1$\bar{1}$1	1$\bar{2}$2

The Operation

Illustration 13 : Let us take the table of 19 as our first example. The operator is -1 for the unit place and +2 for the tenth place.

(i) That means we write the number 19 and divide it into two parts by drawing a vertical line in between the two digits.

(ii) We apply the operator to the digit at the unit place (with appropriate sign - of course). In the present example we apply the operator -1 to the digit 9 (at the unit place), and therefore we get 9-1 = 8 as the second level unit number, 8-1 = 7 as a third level digit, 7-1 = 6 the fourth level, 6-1 = 5 as the fifth level and so on, finally 1-1 = 0 at the tenth level. However, if required, the carry over can be easily done.

This step gives us the digits of the unity column 1

(iii) Next we apply the operator of the tenth place to the digit at the tenth place. Presently we add +2 at every stage to get the tenth place number at the next level. Therefore, we get 1 +2 = 3 as the digit at the second level. Similarly moving further, 3+2 = 5 is the third level digit, 5+2 = 7 is the fourth level digit, 7+2 = 9 the fifth level, 9+2 = 11 is the sixth level digit, 11+2 = 13 as the seventh level digit and so on, 17+2 = 19 at the tenth level.

1	9
	8
	7
	6
	5
	4
	3
	2
	1
	0

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Notes :

1. However it is not essential to stop at tenth level as the same operators can be further applied for continuing to write the table upto hundredth level or even more if so desired.

2. As soon as the operator of addition gives the values larger than 9, the unity at the tenth place of addition (which shall be actually at the hundredth place of the final number) we can leave in the addition process and add only to the single digit left over.

3. The values can be easily checked using Sutra 15.

level		
	2	T
1	1	9
2	3	8
3	5	7
4	7	6
5	9	5
6	11	4
7	13	3
8	15	2
9	17	1
10	19	0

Discussion : We find surprising amount of simplicity in the Vedic approach to this facet of tables. It is always desirable to remember some tables but the Vedic method gives the easy and superfast key to any of the tables and thereby the student is not afraid of the tables. As he is confident that in case of any doubt he can write down the table in a few seconds time and thereby relieves him from the fear and pressure generally felt. And with a relaxed mind even the process of remembering tables becomes easy.

Further any table can be drawn, far beyond the normal range of the mugged up tables, in a matter of seconds for use in any situation where we are required to multiply several times with the same number. (Ref. 1 & 9).

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8. COMPOUND SUBTRACTION

The process of compound subtraction in Vedic Method is converted to a series of simple subtraction and addition. Once again the Sutra Shudha is deployed whenever the large number of a particular unit is to be subtracted from the smaller number of the same unit. It is to be noted that in Vedic Method there is absolutely no need to convert different units to a single unit as is frequently done in the normal procedure. The step by step process is illustrated in the following examples (Ref.2 & 9).

Example 14 :

5 yds	1 ft	9 inches
-2	2	11

Step 1 : The number 11 at inches place is bigger than the corresponding no. 9. Therefore Sudhikaran is required.

(a) We say Shudha and put a dot above the lower number at the next unit (2).

(b) Find the complement of the lower number from the unit size (12) in this case as there are +12 inches in 1 ft.
Therefore 12-11 = 1 is the complement of the lower number 11 (from 1 ft the next unit in this case).

(c) Add this complement to the upper number. 1+9 = 10 is the answer.

5 yd	1 ft	9 inch
-2	$\dot{2}$	11
		10

Step 2 : We move on to next unit (which is ft in this case). Since the lower number has a dot above this, we increase it by 1

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(1) Therefore 2 + 1 = 3 and 3 is larger than the corresponding number at this place (which is 1 in the case). Sudhikaran is required.

(a) We say Sudha and put a dot above the lower number at the next unit (2 yds here).

(b) The complement of this lower number is computed by subtracting it from the unit size, which is 3 in this case (as there are 3 ft in 1 yard). Therefore 3 - 3 = 0 is the complement of the increased lower number 3.

(c) Add this complement to the upper number. Therefore 1 + 0 = 1 is the answer.

5 yd	1 ft	9 inchs
-2	2	11
	1	10

Step 3 : We move on to the next unit which is yards in this case. Since the lower number has a dot above this, we increase it by 1.

Therefore 2 + 1 = 3. In this case the lower number (3) is smaller than the upper number (5) as such 5 - 3 = 2 yards is the answer. Therefore the final answer is 2 yards 1 ft and 10 inches.

NOTE : We observe that in the Vedic Method of compound subtraction there is no carry over of the unit instead we only find out the complement of each number. The later being easier, hastens the computation process and also reduces the chances of an error.

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9. COMPOUND ADDITION

Introduction

The process of compound addition in Vedic Method is very similar to the compound subtraction. The Sutra invoked is also the same, namely Shudha. The process of Shudhikaran is done whenever the addition of numbers at a particular unit exceeds the unit size. The steps involved are similar to those in subtraction. The basic differences are (i) the complement is computed for the larger number (ii) the complement is subtracted from the smaller number to get the answer.

Illustration 15 :

4 yards	1 ft	11 inches
+3 yards	2 ft	8 inches

Step I : The larger number is 11 inches. The process of shudhikaran is carried out, as it is very close to the unit size (12 inches) in this case.

(a) We say Shudha and put a dot above the larger number in the next unit (2 ft).

(b) Compute the complement of the larger number (12-11 = 1).

(c) Subtract this component from the smaller number, 8-1 = 7 is the answer.

4 yards	1 ft	11 incheas
3 yards	2 ft	8 inches
		7

Step II : We move on to the next unit (here ft). Since the larger number has a dot above this, we increase it by 1, 2+1 = 3 ft. The process of Shudhikaran is carried out.

(a) We say Shudha and put a dot above the larger number at the next unit place.

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(b) Compute the complement of the larger number. 3-3 = 0 the complement when subtracted from the smaller number (1 ft) gives the answer, 1-0 = 1 ft.

$\dot{4}$ yards	1 ft	11 inches
+3	$\dot{2}$	8
	1	7

Step III : We move to the next unit, which is yards in this case. The larger number (4) has a dot above it. Therefore, it is increased by 1(4 + 1 = 5) and added to the smaller number 5 + 3 = 8 yards is the answer. The process of Shudhikaran is not required as this unit has a very large unit size.

4 yards	1 ft	11 inches
+3	2	8
8	1	7

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10. VEDIC CHECK METHOD

Checking of Compound Subtraction (Ref. 2 & 9)

First of all let us consider the cases of compound subtraction. In addition to the steps already considered for checking the ordinary subtraction we have to take care of the unit size and also the dot due to Shudhikaran at each unit place. Further the checking can be conveniently carried on for each unit separately.

Example 16 :

Let us understand the step by step process with the help of following example

5 yards	1 ft.	9 inches
-2	2	11
2	1	10

The compound subtraction can be carried on by using the Śudhikaran process. We observed that the dot of Shudhikaran has appeared in the feet as well as yards units because of the larger numbers being subtracted at the inch and feet places, respectively. The check is applied separately for each unit.

STAGE I : Let us first take the inch unit.

Step 1 : The result 10 is converted to a single-digit number, 1+0 = 1.

Step 2 : The number subtracted (11) is converted to a single-digit number, 1+1 = 2. Further if we observed the dot of Shudhikaran appearing on this number we increase it by 1.

Step 3 : Now we add the single-digit of the result to the equivalent single digit of the number subtracted as obtained in step 1 and 2.

Step 4 : Similarly the original number is also converted to a single-digit number.

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Here it is just 9. We observed the number at higher unit. If it has a dot of Shudhikaran we shall also add the unit size (12) to the original number and of course we shall reduce it to the single digit form. In the present case, we find the Shudhikaran for over the number 2 in the next unit feet. Therefore, we add the unit size (1 feet = 12 inches) to the original number. Therefore 9+12 = 21, 2+1 = 3. In the present example we get 3.

Step 5 : The check - If our compound subtraction has been correctly done, we should get the same single-digit numbers in steps 3 and 4 respectively. In the present example we have obtained 3. This is a reasonable proof of the correctness of the operation at the inch unit.

STAGE II : After checking the computations at inch unit we repeat the same steps for the next unit.

Step 1 : The result is just 1.

Step 2 : The number subtracted is just 2. But since we observe a Shudhikaran dot above 2, we increase it by 1. Therefore 2 + 1 = 3.

Step 3 : Now we add the single digit of the result to the equivalent single digit of the number subtracted as obtained in steps 1 and 2. In the present case 3+1 = 4.

Step 4 : Once again the original number is just 1. Further we observed that there is a Shudhikaran dot above the number subtracted at the next unit, namely, yards. Therefore we add the unit size (1 yard = 3 feet) to the original number. In the present case we get 1+3 = 4.

Step 5 : The check - We compare the numbers obtained in step 3 and 4 to ascertain the correctness. Here we have obtained the same number 4.

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STAGE III : Now we come to the unit yards.

Step 1 : The result is just 2.

Step 2 : The subtracted number is also 2. But since it has a 'Shudhikaran dot' we increase it by 1, and get 2+1 = 3.

Step 3 : The addition of the single digits of the results and the number subtracted gives 2+3 = 5.

Step 4 : The original number is just 5.

Step 5 : On comparing the single digit numbers in step 3 and 4 we find that both are 5 each. Hence the computations are correct.

Compound Addition Check

Like the previously discussed check methods it is possible to check the correctness of any compound addition also. It is interesting to mention that the single Vedic sutra is at work in all these checks. For compound addition also, we apply the check separately for each unit and hence to keep track of the presence of the 'Shudhikaran dot'. Further the unit size shall be frequently used in this process.

11. MULTIPLICATION : Nikhilam Method :

It is interesting to note that there is a general formula which can be applied to all cases of multiplication. This formula is quite easy and simple. However, there are special cases where special Vedic procedure gives much simpler, easier and quicker solutions. The latter shall be discussed first. (Ref. 1,2,4 & 9).

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The second sutra reads 'Nikhilam Navatascaraman Da satah'. The literal translation of this crisp Sutra "निखिलं नवतश्चरमं दशतः" is

'All from 9 and the last from 10.

Now the step by step procedure is outlined below :

TYPE-I

Let us take the simple example of multiplying 13 by 11.

Step 1 : We write these numbers in the Vedic base system style and get :

$$\begin{array}{l} 13 + 3 \\ 11 + 1 \end{array}$$

We note that the base is 10 and there is only1zero in the base. Positive sign indicates that both numbers are greater than the base.

Step 2 : The product will have2parts. A vertical dividing line should be drawn to demarcate these two parts.

$$\begin{array}{l|l} 13 + 1 & \\ 11 + 1 & \\ \hline & 1 \end{array}$$

Step 3 : We vertically multiply the excess digits from base. This product gives us the right hand side portion of the answer. In this case 3 x 1 = 3 is the product.

$$\begin{array}{l|l} 13 + 3 & \\ 11 + 1 & \\ \hline & 13 \end{array}$$

Step 4 : The left hand side of the answer can be arrived at by simple adding diagonally any of the two given numbers with the deviation (deficit) of the other number from base.

Case 1 : The first number 13 is added to 1, the deficit of second number. Therefore

$$13 + 1 = 14$$

Case 2 : The second number 11 is added to 3 the deficit of the first number. Therefore

$$11 + 3 = 14$$

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Note : Both of these alternatives gives the same result

$$\begin{array}{l} 13 + 3 \\ 11 + 1 \\ \hline 14 \,/\, 3 \end{array}$$

In addition to the above two alternatives the left hand side can also be obtained by two additional procedures.

Note :

The Vedic System has this special feature of availability of the same result in several easy ways. This feature can be utilised by the student to verify his answer step by step.

Further, **Illustration 17 :**

$$\begin{array}{l} 111 \times 112 \\ 111 + 11 \\ 112 + 12 \\ \hline 123 \,/\, {}_{1}32 \end{array} = 123+1/32 = 124/32 = 12432$$

Note : The R.H.S. should be obtained mentally by using Vedic method i.e.

$$\begin{array}{l} 11 + 1 \\ 12 + 2 \\ \hline 13 \,/\, 2 \end{array}$$

$$\begin{array}{l} 10010 \times 10101 \\ 10010 + 10 \\ 10101 + 101 \\ \hline 10111/1010 \end{array}$$

$$\begin{array}{l} 1099 \times 1001 \\ 1099 + 99 \\ 1001 + 01 \\ \hline 1100 \,/\, 099 \end{array}$$

$$\begin{array}{l} 103 \times 13 \\ \text{Taking base as } 10 \\ 103 + 93 \\ 13 + 3 \\ \hline 106 \,/\, {}_{27}9 \end{array} = 106+27/9 = 133/9 = 1339$$

$$\begin{array}{l} 985 \times 995 \\ 985 - 15 \\ 995 - 5 \\ \hline 980 \,/\, 075 \end{array}$$

$$\begin{array}{l} 99976 \times 99998 \\ 99976 - 24 \\ 99998 - 2 \\ \hline 00074/00048 \end{array}$$

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$$\begin{array}{r} 103 + 3 \\ 97 - 3 \\ \hline 100 \,/\, 0\bar{9} \end{array} = 99/100\text{-}09 = 9991$$

$$\begin{array}{r} 1021 + 21 \\ 996 - 4 \\ \hline 1017 \,/\, 0\bar{8}\bar{4} \end{array} = 1016/1000\text{-}84 = 1016916$$

Note : The right hand portion of the answer is with a -ve sign, called the 'Vinculum-portion'. Even the subtraction of the Vinculum portion can be easily done with the aid of the 'Nikhilam Sutra'.

Multiples & Sub-Multiples : In all the examples discussed in the previous section we have observed that both the multiplicand and the multiplier or at least one of them is very close to the base taken. However, frequently we have to multiply two numbers neither of which is near a convenient base. The solution of these problems can be obtained by using a small 'Upa sutra'; 'Anurupyena' which means proportionately, when applied, it means in all cases where there is a rational relationship (ratio), the ratio should be taken into account. This 'ratio' shall be lead to a proportionate multiplication or division as the case may be. (Ref. 2,3 & 4).

Illustration 18 : Case 1 : Multiply 43 x 43

Both of these numbers are away from the base 100, we get the deficiencies as 57 and 57 respectively and the vertical multiplication of so large numbers is inadmissible in Vedic system. In these cases we accept 100 as a theoretical base and the submultiple 50 (which is near 43 & 43) as our working base (WB). The remaining procedure is outlined below :

Step 1 : Base 100

Working base, WB = 50

Submultiple = 1/2 because $100 \times \frac{1}{2} = 50$ (WB)

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Step 2 : The deviations of the numbers from the sub-base are 7 and 7 respectively. Therefore, according to Vedic style we write

Step 3 : The R.H.S. is 07x 07 = 49

$$\begin{array}{r} 43 - 07 \\ 43 - 07 \\ \hline 18 \,/\, 49 \end{array}$$

Note : Since our base is 100, two digits are admissible on R.H.S.

Step 4 : By cross subtraction we get 43 - 07 = 36 as our L.H.S.

Step 5 : Now since the sub-multiple is $\frac{1}{2}$ we multiply the L.H.S. with the submultiple i.e. $\frac{1}{2}$ x 36 = 18. The resulting number is the LHS portion of the answer.

Case 2 : Alternately, we may take 10 as our base and 40 as the working base. The multiple is 4 (4 x 10 = 40)

$$\begin{array}{r} 43 + 3 \\ 43 + 3 \\ \hline 46 \,/\, 9 \end{array}$$

The L.H.S. is 46 x 4 = 184

$$\begin{array}{r} 43 + 3 \\ 43 + 3 \\ \hline 184 \,/\, 9 \end{array}$$

There 1849 is the answer.

COMMENT : We get the same answer.

Note : The above example clearly explains the use of Upasutra 'Anurupyena'. The result obtained by base is proportionatly modified to get the final solution.

Illustration 19 :

Multiply 252 x 254

Base	1000
Working base	250
Submultiple	$\frac{1}{4}$ (because 1000 x $\frac{1}{4}$ = 250)

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252 + 002
254 + 004
---
256 / 008
x
1/4
= 64/008 = 64008

12. NIKHILAM COROLLARIES

The previously illustrated examples were all solved by using the Nikhilam Sutra.

However, there are a few more corollaries arising out of the Nikhilam Sutra. We shall now learn these corollaries . (Ref. 1,2,4 & 9).

The First Corollary : The first corollary of Nikhilam Sutra is Yavadunam Tavadunikrtya Varganca Yojayet. It means what ever the extent of its deficiency lessen it further to that very extent; and also set up the square (of that deficiency).

This sutra essentially deals with squaring of the numbers.

Illustration 20 :

92^2 = (92 - 8)/8^2 = 84/64 = 8464
94^2 = (94 - 6)/6^2 = 88/36 = 8836
95^2 = (95 - 5)/5^2 = 90/25 = 9025
96^2 = (96 - 4)/4^2 = 92/16 = 9216
97^2 = (97 - 3)/03^2 = 94/09 = 9409
989^2 = (989-11)/011^2 = 978/121 = 978121
9989^2 = (9989-0011)/0011^2 = 9978/0121 = 99780121
9993^2 = 9986/0049 = 99860049
112^2 = 112+12/12^2 = 124/144 = 12544
1011^2 = 1011+11/011^2 = 1022/121 = 1022/121 = 1022121

COMMENT : The algebraic proof of these methods has also been presented by Jagatguru Swami Sri Bharti Krisna Tirthaji Maharaja in his book on Vedic Mathematics.

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The Second Corollary : The second corollary is the Sutra Ekadhikena Purvena. This sutra is used as a subsutra in the present case. This is a special case of the first corollary, and is applicable for squaring the number having 5 as the last digit. It's meaning is by one more than the previous one.

Square of 35 : Here the previous digit is 3. The sutra in this context tells us to multiply the previous digit (here 3) by one more than the previous number (3 + 1 = 4). Therefore, 3 x 4 = 12 is L.H.S. portion of the answer. The R.H.S. is simply the vertical multiplication product (5 x 5 = 25). Therefore 35^2 = 3 x 4/5 x 5 = 1225.

Illustration 21 :

65^2 = 6 x 7/25 = 42/25 = 4225

95^2 = 9 x 10/25 = 90/25 = 9025

115^2 = 11 x 12/25 = 132/25 = 13225

135^2 = 13 x 14/25 = 13 + 3 / 14 + 4 / 25 = 18225
18 /2 /

195^2 = 19 x 20/25 = 38025

155^2 = 15 x 16/25 = 240/25 = 24025

The Third Corollary : In mathematical astronomy we frequently require the multiplication of multiplier digits consisting entirely of nines with other numbers (multiplicand). For multiplication of this type the subsutra Ekanyunena-Purvena (Sutra No. 14) is used.

Multiply 6 x 9

According to the sutra the multiplicand has to be decreased by 1 (this gives the LHS) and for the R.H.S. subtract the L.H.S. from the multiplier. (Get compliments of nines). Therefore 6 x 9 = (6-1)/9-(6-1) = 5/4 = 54

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Illustration 22 :

19 x 99 = 18/(99-18) = 18/81 = 1881

778 x 999 = 777/222 = 777222

9863 x 999 = 9862/0137 = 98620137

3728387x9999999 = 3728386/6271613 = 37283866271613

If the multiplicand is having a smaller number of digits than the multiplier, the Sutra Ekanyunena is equally applicable. It is necessary to fill in the blanks on the left with the required number of zeros. The remaining process is unaltered.

Illustration 23 :

67 x 999 = 067 x 999 = 066/933 = 66933

737x99999 = 00737 x 99999 = 00736/99263 = 829999917

83x9999999 = 0000083x9999999 = 0000082/9999917

If the multiplier consists of smaller number of nines following the above procedure, the answer is obtained.

Illustration 24 :

17 x 9 = 16/9-16 = $16/\bar{7}$ = 15/10-7 = 15/3 = 153

163 x 9 = 162/9 - 162

= $162/\bar{1}\bar{5}^{\bar{3}}$ = $162 - 15/\bar{3}$

= $147/\bar{3}$ = 146/10 $-\bar{3}$ = 1467

COMMENTS : This case yields vinculum on the R.H.S.

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13. OPERATION OF VINCULUM NUMBERS - PART I

Addition of Vinculum Numbers (Part I)

The addition of numbers in Vinculum form is similar to the ordinary addition the only difference being that each digit is operated alongwith its sign. The following examples shall explain the point (Ref. 3 & 9).

ILLUSTRATION 25 : We want to add

$$\begin{array}{rr} & 2\,\bar{3}\,4\,\bar{2} \\ + & 4\,\bar{2}\,\bar{1}\,3 \\ \hline \end{array}$$

Step 1 : At the unit place 3 is to be added to $\bar{2}$ resulting in $3+\bar{2} = 3-2 = 1$.

Step 2 : $\bar{1}$ is to be added to 4 at the tenth place therefore $4+\bar{1} = 4-1 = 3$.

Step 3 : $\bar{2}$ is to be added to $\bar{3}$ at the hundredth place giving us $\bar{2}+\bar{3} = -2-3 = -5 = \bar{5}$.

Step 4 : 4 is added to 2 at the thousandth place. This gives 4+2 = 6. Therefore we get

$$\begin{array}{rr} & 2\,\bar{3}\,4\,\bar{2} \\ & 4\,\bar{2}\,\bar{1}\,3 \\ \hline & 6\,\bar{5}\,3\,1 \end{array} \quad = \quad 5531$$

Subtraction of Vinculum Numbers (Part I)

The subtraction of numbers in Vinculum form is similar to the ordinary subtraction, the only difference being that each digit is operated along with its sign. The following examples shall explain the point.

ILLUSTRATION 26 :

1. $4 - 3 = 1$
2. $3 - 4 = -1 = \bar{1}$

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3. $4 - \bar{2} = 4 - (-2) = 4 + 2 = 6$

4. $\bar{3} - 2 = -3 - 2 = -5 = \bar{5}$

5. $\bar{2} - \bar{3} = -2 - (-3) = -2 + 3 = 1$

6. $\bar{4} - \bar{2} = -4 - (-2) = -4 + 2 = -2 = \bar{2}$

The subtraction of the numbers in the Vinculum form shall consist of only one of the above six cases. The subtraction of individual digits can be carried out without any borrowing process as shown above in case 2,4,5 and 6.

ILLUSTRATION 27 :

$$\begin{array}{r} 5\,\bar{4}\,2\,\bar{3}\,1 \\ -\,2\,\bar{2}\,3\,\bar{4}\,3 \\ \hline 3\,\bar{2}\,\bar{1}\,1\,\bar{2} \end{array}$$

Step 1 : $1 - 3 = -2 = \bar{2}$ at the unit place

Step 2 : $\bar{3} - \bar{4} = -3-(-4) = -3 +4 = 1$ at tenth place.

Step 3 : $2 - 3 = -1 = \bar{1}$ at hundredth place

Step 4 : $\bar{4} - \bar{2} = -4 - (-2) = -4 + 2 = -2 = \bar{2}$ at thousandth place

Step 5 : $5 - 2 = 3$. Therefore we get the answer as $3\,\bar{2}\,\bar{1}\,1\,\bar{2}$, which in the normalised form gives 2 7 9 0 8.

Multiplication of Vinculum Digits

The product of digits in the Vinculum form is done by using the simple algebraic rules.

ILLUSTRATION 28 :

1. $3 \times 2 = 6$
2. $\bar{2} \times 4 = -2 \times 4 = -8 = \bar{8}$
3. $\bar{3} \times \bar{2} = -3 \times (-2) = 6$
4. $2 \times \bar{3} = 2 \times (-3) = -6 = \bar{6}$
5. $\bar{4} \times 3 = -4 \times 3 = -12 = \bar{1}\bar{2}$
6. $4 \times \bar{5} = 4 \times (-5) = -20 = \bar{2}\bar{0}$
7. $\bar{3} \times \bar{4} = -3 \times (-4) = 12$

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14. OPERATIONS OF VINCULUM NUMBERS (PART II)

Addition of Vinculum Numbers (Part II) (Ref. 3 & 9)

While adding the Vinculum numbers the carryover may be sometimes required. This is done in the usual Vedic style by using the Shudha Sutra. However in operation of Vinculum numbers two types of situations may arise. The first, when the addition of positive digits results in a number greater than 9. We say 'Shudha' and put a dot over the last digit just added, drop the unity at the tenth place and retain the digit at the unit place for subsequent operation. The second case arises when the sum of the negative digits (that is the digits with a bar) results in a number greater than 9. Once again we say Shudha, put a dot over the last bar digit just added, drop the unity at the tenth place, and retain the digit at the unit place for further operations. This digit is of course with a negative sign (and shall be written with a bar). Further, this is to be noted that as each dot of Shudhikaran over a positive number is carried over as 1 in the next column (place), similarly each dot over a bar digit (which is essentially a negative digit) is carried over as unity with a bar to the next place. The following illustrations shall explain the process.

ILLUSTRATION 29 :

$$\begin{array}{r} 4\,\bar{3}\,\dot{5}\,\bar{4} \\ +\,3\,\bar{4}\,6\,2 \\ \hline 7\,\bar{6}\,1\,\bar{2} \end{array} \quad = 6408$$

Step 1 : $-4 + 2 = -2$ or $\bar{2}$

Step 2 : $5 + 6 = 11$ (Shudhikaran) and a dot over 5,1 is the answer.

Step 3 : (a) $-3 + 1 = -2$ (due to the Shudhikaran dot on 5)

(b) $-4 + (-2) = -6$ or $\bar{6}$

Step 4 : $4 + 3 = 7$. Therefore we get, $7\,\bar{6}\,1\,\bar{2}$, in normalised form 6408.

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ILLUSTRATION 30 :

$$\begin{array}{r} 2\dot{\bar{5}}\dot{\bar{5}}4 \\ +\ 4\bar{4}\bar{5}\bar{5} \\ \hline 5001\bar{1} \end{array}$$

Step 1 : $4 + (-5) = -1$

Step 2 : $\bar{5} + \bar{5} = \bar{1}\bar{0}$ (Shudhikaran)

Step 3 : $\bar{5} + \bar{1} = \bar{6}$

$\bar{6} + \bar{4} = \bar{1}\bar{0}$ (Shudhikaran); 0 retained

Step 4 : $2 + \bar{1} = 1$

$4 + 1 = 5$. Therefore $500\bar{1}$, in normalise form 4999.

Subtraction of Vinculum Numbers (Part II) (Ref. 3 & 9)

While carrying on the subtraction of Vinculum numbers no borrowing may be required but instead a negative carryover may be required this shall be done by using the 'Sutra Shudha' as already explained in the previous section.

ILLUSTRATION 31 :

$$\begin{array}{r} 56\bar{4}\dot{8}3\bar{5} \\ -\ 4\bar{2}\bar{5}\bar{5}\bar{2}4 \\ \hline 18235\bar{9} \end{array}$$

Step 1 : $\bar{5} - 4 = -5 - 4 = \bar{9}$

Step 2 : At tenth place $3 - \bar{2} = 3 + 2 = 5$

Step 3 : $8 - \bar{5} = 8 + 5 = 13$ so we say 'Shudha' and carry over 1 to the next digit and retain 3 at the hundredth place.

Step 4 : Since 1 is to be carried over $\bar{5}$ so $1 - \bar{5} = 1+5 = 6$ and $6 + \bar{4} = 6 - 4 = 2$.

Step 5 : $6 - \bar{2} = 6 + 2 = 8$

Step 6 : $5 - 4 = 1$

$$\begin{array}{r} 56\bar{4}\dot{8}3\bar{5} \\ -\ 4\bar{2}\bar{5}\bar{5}\bar{2}4 \\ \hline 18235\bar{9} \end{array} = 182341 \text{ Ans.}$$

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15. CHECKING VINCULUM NUMBER OPERATIONS

Addition Check (Ref. 3 & 9)

The steps required for checking the correctness of any addition of Vinculum numbers, are the same as those for simple addition.

We convert each number to an equivalent single digit number, and operate the single digit numbers. This should be equal to the single digit form of result.

ILLUSTRATION 32 :

$$\begin{array}{r} 4\,\bar{2}\,\dot{\bar{5}}\,3 \\ +\ 5\,3\,\bar{5}\,\bar{4} \\ \hline 9\,0\,0\,\bar{1} \end{array}$$

Step 1 : Add the first number to get the single digit number $4+(-2) = 2$, $2+\bar{5} = \bar{3}$, $\bar{3}+3 = 0$.

Step 2 : Similarly $5+3 = 8$, $8+(-5) = 3$, $3+(-4) = -1$ or $\bar{1}$.

Step 3 : Now add the two numbers obtained $\bar{1} + 0 = \bar{1}$

Step 4 : The result : $9 + 0 = 9$, $9 + 0 = 9$, $9 + (-1) = 8$.

Step 5 : NOTE : Here 9 is the turn point digit in checking system and not 10, because $10 = 1 + 0 = 1$. Therefore $\bar{1} \equiv 9 - 1 \equiv 8$.

Hence 8 = 8 Okay

Subtraction

Like addition, the steps for checking the subtraction of Vinculum numbers are similar to simple number subtraction. The additional points are brought out through the illustrative examples.

ILLUSTRATION 33 :

$$\begin{array}{r} 5\,\dot{\bar{4}}\,\dot{3}\,2\,\dot{\bar{7}} \\ -\ 2\,\bar{5}\,\bar{9}\,\bar{5}\,4 \\ \hline 3\,2\,2\,6\,\bar{1} \end{array}$$

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Step 1 : $5 + \bar{4} = 1, 3 + 1 = 4, 4 + 2 = 6, 6 + \bar{7} = \bar{1}.$

Step 2 : $2 + \bar{5} = \bar{3}, \bar{9} + \bar{3} = \bar{12}; \bar{1} + \bar{2} = \bar{3}, \bar{5} + \bar{3} = \bar{8}, \bar{8} + 4 = \bar{4}$

Step 3 : $3 + 2 = 5, 5 + 2 = 7, 7 + 6 = 13, 1 + 3 = 4, 4 + \bar{1} = 3.$

Step 4 : $\bar{4} + 3 = \bar{1}$ (Add result of step 2 and step 3)

Step 5 : Hence $\bar{1} = \bar{1}$ Okay. (same values at steps 1 & 4).

Checking the Vinculum Numbers

It is interesting to note that this simple and straight forward Vedic Check Method for ascertaining the absence of any random error, can also be applied for checking the correctness of the numbers after they are converted to the Vinculum form and vice versa.

The simple procedure is just the same, with the additional facet of turning point at 9 and the corresponding equivalence of the positive and the complementary negative digits. The following example shall simply illustrate the whole process.

ILLUSTRATION 34 :

The number 46313 in Vinculum form is = $5\ \bar{4}\ 3\ 1\ 3$

Single digit equivalent $4 + 6 = 10, 1 + 0 = 1, 1 + 3 = 4, 4 + 1 = 5, 5 + 3 = 8.$

The Vinculum number similarly gives $5 + \bar{4} = 1, 1 + 3 = 4, 4 + 1 = 5, 5 + 3 = 8.$

We get the same single digit number (8) for both the normal and the Vinculum number. This is an indicator that there is no random error.

ILLUSTRATION 35 :

The transformation of the Vinculum number $2\ \bar{5}\ \bar{9}\ \bar{5}\ 4$ into the normalised form gives 1 4 0 5 4 .

The Vinculum number gives $2 + \bar{5} = \bar{3}$; we ignore

$\bar{9}; \bar{3} + \bar{5} = \bar{8}; \bar{8} + 4 = \bar{4}.$

The number gives $1 + 4 = 5; 5 + 5 = 10; 1 + 0 = 1; 1 + 4 = 5$

But we know that $\bar{4} \equiv 9 - 4 = 5$. Hence the check is satisfied.

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16. THE GENERAL METHOD OF MULTIPLICATION

So far we have seen the use of the Vedic Sutras for the solution of specific types of problems. Now we shall learn the simple Vedic general method of multiplication. This method is based on the Urdh va-Tiryagbhyam Sutra. The simple meaning of this Sutra is 'Vertically and Crosswise'. But this Sutra is very powerful and applicable for solving a large class of problems in mathematics. Further, it is interesting to point out that this brief Sutra has many fold applications. Prof. Nicholas, Pickles and Prof. Williams have written a full book on applications of this Sutra (see VM news). The step by step process is outlined in the following sections (Ref. 1,2,3,4 & 9).

SIMPLIFICATION

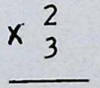
For understanding the application of Urdhva Sutra it is important to first learn the method of simplification of different number of columns.

Level 1 : If there are only 2 digits in a single column we just multiply them Vertically to get R the resultant number.

$$R = U_1 \times L_1$$

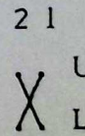
For example

$$\therefore \quad R = 2 \times 3 = 6$$



If however the resulting number consists of more than one digit, the extra digit at tenth place written below the next higher space.

Level 2 : If the simplification consists of two columns with four digits, the resulting number is obtained by cross multiplication of the diagonally opposite digits and algebraic addition of the products.



$$R = U_1 \times L_2 + U_2 \times L_1$$

U is the upper row
L is the lower row and U_1 and U_2 are the digits in the upper row in the 1st and 2nd column respectively. Similarly L_1 and L_2 are the digits of the lower row in column 1 and 2 respectively (From right).

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Illustration 36 : Let us consider the multiplication of 2 by 2 digit numbers.

Multiply 13 x 21

$$\begin{array}{r} 21 \\ \times\ 13 \\ \hline 3 \\ \hline \end{array}$$

Step 1 : We simplify the first column

$$R_1 = U_1 \times L_1$$
$$= 1 \times 3 = 3$$

$R_1 = 3$ is the extreme right part of the answer and is written below the first column.

Step 2 : We operate the first column with each of the subsequent column. We include the second column which on simplification gives the next digit

$$R_2 = U_1 \times L_2 + U_2 \times L_1$$
$$= 1 \times 1 + 2 \times 3 = 7$$

$R_2 = 7$ is the result at the next level and is written at the tenth level.

With this we complete the operation of the first column.

Step 3 : We eliminate the first column and simplify the remaining columns (here only one column is left) therefore

$$R_3 = U_2 \times L_2$$
$$= 2 \times 1 = 2$$

$R_3 = 2$ is the resulting digit at the next level (hundred in this case). The result is written into appropriate level.

$$\begin{array}{r} 21 \\ \times\ 13 \\ \hline 2/7/3 \end{array} = 273$$

Illustration 37 :

$$\begin{array}{cc} 2 & 1 \\ \times & \begin{matrix} U \\ L \end{matrix} \end{array}$$

Multiply 79 by 68

$$\begin{array}{r} 79 \\ 68 \\ \hline 7\ 2 \end{array}$$

Simplify the first column

$R1 = U1 \times L1 = 8 \times 9 = 72$

$R1 = 72$ is the extreme right part of the answer, consisting of two digits. Therefore, we write 2 below the first column and excess digit 7 below the next space.

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Operation of the first column with the second column gives us the next part of the product

$R2 = U1 \times L2 + U2 \times L1$

$= 9 \times 6 + 7 \times 8 = 54 + 56 = 110$

Once again we enter 0 at the next space and 11 below the subsequent spaces.

$$\begin{array}{r} 7\ 9 \\ 6\ 8 \\ \hline 0\ 2 \\ 11\ 7 \end{array}$$

With this we complete the operation of the first colunm. After operating the first column with all other columns we start eliminating, the column one by one.

Eliminating first column, we are left with only the second column. Therefore $R3 = U_2 \times L_2 = 7 \times 6 = 42$ is the last part of the answer. We enter this value and add the carry over digits to get the final answer

$$\begin{array}{r} 7\ \ 9 \\ 6\ \ 8 \\ \hline 4\ 2\ 0\ 2 \\ 1\ 1\ 7 \\ \hline 5\ 3\ 7\ 2 \end{array}$$

Vinculum : Alternatively we may convert this number into the Vinculum form and then multiply

$79 = 1\ \bar{2}\ \bar{1}$ and $68 = 1\ \bar{3}\ \bar{2}$

$$\begin{array}{r} 1\ \bar{2}\ \bar{1} \\ 1\ \bar{3}\ \bar{2} \\ \hline 3\,/\,7\,/\,2 \end{array}$$

$R1 = U_1 \times L_1 = \bar{1} \times \bar{2} = 2$

$R2 = U_1 \times L_2 + U_2 \times L_1 = \bar{1} \times \bar{3} + \bar{2} \times \bar{2}$

$= 3 + 4 = 7$

$R3 = U_1 \times L_3 + U_3 \times L_1 + U_2 \times L_2$

$= \bar{1} \times 1 + 1 \times \bar{2} + \bar{2} \times \bar{3}$

$= \bar{1} + \bar{2} + 6$

$= 3$

With this we complete the operation of the first column with the remaining columns. For subsequent parts of the answer we start eliminating the columns, one

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by one. Eliminating the first column we simplify the remaining column.

Therefore, R4 = $U_2 \times L_3 + U_3 \times L_2$

= $\bar{2} \times 1 + 1 \times \bar{3}$

= $\bar{2} + \bar{3} = \bar{5}$

and R5 = $U_3 \times L_3 = 1 \times 1 = 1$

$$\begin{array}{r} 1\ \ \bar{2}\ \ \bar{1} \\ 1\ \ \bar{3}\ \ \bar{2} \\ \hline 1/\bar{5}/3/7\ /2 \end{array}$$

$= 1\bar{5}372 = 5372$

Symbolically we can say that

$$R1 = \begin{matrix} 1 & \bar{2} \\ 1 & \bar{3} \end{matrix}\left[\begin{matrix} \bar{1} \\ \bar{2} \end{matrix}\right] \quad ; \quad R2 = 1\left[\begin{matrix} \bar{2} & \bar{1} \\ \bar{3} & \bar{2} \end{matrix}\right]$$

$$R3 = \left[\begin{matrix} 1 & \bar{2} & \bar{1} \\ 1 & \bar{3} & \bar{2} \end{matrix}\right] \quad ; \quad R4 = \left[\begin{matrix} 1 & \bar{2} \\ 1 & \bar{3} \end{matrix}\right]\begin{matrix} \bar{1} \\ \bar{2} \end{matrix}$$

$$R5 = \left[\begin{matrix} 1 \\ 1 \end{matrix}\right]\begin{matrix} \bar{2} & \bar{1} \\ \bar{3} & \bar{2} \end{matrix}$$

For each part we simplify the columnswritten within the brackets.

Notes : (1) We clearly observe that the arithmetical work involved is greatly simplified by using Vinculum approach.

(2) The whole process can be carried out mentally after a little practice.

Direct Compound Multiplication

In the Vedic method, we can do the compound multiplication directly by using the 'Urdh vaSutra'.(Ref. 1,3,4 & 9).

Illustration 38: Let us say we want to find the area of a plot of size 4 yards 1 feet by 8 yards 2 feet.

$$\begin{array}{ll} 4\text{ yd} & 1\text{ feet} \\ \underline{8\text{ yd}} & \underline{2\text{ feet}} \\ 32\ /\ 16\ /\ 2 & \end{array}$$

Step I : We use the Urdh va Sutra (No. 3) to obtain three parts of the answer.

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(1) R.H.S = vertical product of RHS column = 2 x 1 = 2

(2) Middle part = cross multiplication = 1 x 8 + 2 x 4 = 16

This is entered in the middle portion.

(3) L.H.S. = vertical product of LHS column = 4 x 8 = 32

Step II : Now we split the middle part into two. First part consists of that number which when divided by the unit size gives the maximum whole number quotient (here 1 yard = 3 feet, therefore 3 is the unit size). And the second part is the remainder which is always less than the unit size.

In the present example 5 x 3 = 15 is the first part (which gives the maximum whole number quotient = 5) and 16 - 15 = 1 is the second part 32/16/2 = 32/5x3+1/2

Step III : In the first part we drop the unit size (here 3) and carry the multiplier (here 5) to LHS and add it to the value already there.

Here, we carry 5, therefore 32+5 = 37 square yards.

The second part (which is the remainder) is multiplied by unit size and the product carried to RHS. This is added to the value already there.

Here 1x3 = 3 is carried to RHS and added to the previous value = 2, which is already there. Therefore 2+3 = 5 square feet is the answer.

32 / 5x3 + 1 /2 = 32+5/ /1x3 + 2 = 37 square yards / 5 square feet

This example explains the simple Vedic method of compound multiplication without the use of any conversion which is essential in present method.

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17. MULTIPLE CHOICE

Vedic Mathematics provides multiple choice for almost every thing which we have to do. This makes Maths a playful and lively subject which children learn with smiles.

Even while handling numbers we have a choice to use the numbers or their complements.

For example $9 = 10-1 = 1\bar{1}$

$8 = 10-2 = 1\bar{2}$

$19 = 20-1 = 2\bar{1}$ etc.

Further, if instead of using 19 we use $2\bar{1}$, which is far more easier, simpler, faster and has lesser chances of error.

ADDITION : For adding 8 to 9 we can either add in the usual way or we can add 8 to the complement of $9(1\bar{1})$ and get $8+1 = 7$ at the unit place, and 17 is the result. Similarily for adding 7 to 8, in addition to the normal way we can also add 7 to the complement of $8(1\bar{2})$. At unit place $7+2 = 5$ and 15 is the result. Small children may find it simpler to reduce 8 by 1 than to add 8 to 9. (Ref. item 1 & 13).

SUBTRACTION : This can be simply looked as a simple addition of the first number with bar put on all the digits of the number to be subtracted. Again as already illustrated the process of Shudhikaran can be avoided most of the times by computing the result in the vinculum form. (Ref. item 2 & 13). Further details are available in Pushp-2.

$$\begin{array}{r} 3\ 2\ 7\ 4 \\ -\ 1\ 4\ 5\ 5 \\ \hline 2\ \bar{2}\ 2\ \bar{1} \\ \hline \end{array} \quad = \quad 1819$$

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MULTIPLICATION : Even for the simple problem of multiplying the two numbers, Vedic Maths provides a large number of alternative ways of computing the result. This makes Maths a playful subject. Students always like to play games and anybody with whom we play games, we naturally become friendly. And there is no fear or anxiety with a friend.

Example 39 : Multiply 89 x 99

This typical example can be solved in seven different ways:

(i) The easiest solution can be obtained by applying Ekanuyena sutra. (Ref. item 12)

$$89 \times 99 = 88-1/99-88$$
$$= 88/11 = 8811.$$

(ii) We can use the Nikhilam method with ease and simplicity. (Ref. item 11)

$$\begin{array}{r} 89 - 11 \\ 99 - 01 \\ \hline 88 \,/\, 11 \end{array} \quad = 8811$$

Even here the L.H.S. can be obtained in four different ways.

(iii) We can use the Urdhva sutra and compute the results from right to left. (Ref. item 16)

$$\begin{array}{r} 89 \\ 99 \\ \hline 88 \,/\, 16^{1} \,/\, 8^{1} \end{array} \quad = 8811$$

This is the most difficult way and a student of Vedic Maths shall never use this method. But it is interesting to point out that this method is closest to the only method being taught and used by all of us.

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(iv) The Urdhva sutra can also be used to compute the result from left to right in two steps.

$$\begin{array}{r} 89 \\ 99 \\ \hline 72\,/\,15^{3}/8^{1} \end{array} = 8811$$

After a little practice even left to right computations can be done in one line.

(v) It will be much more convenient and easy to first convert the given numbers in their vinculum form and then apply the Urdhva sutra. (Ref. item 6 & 16). Further details are available in Pushp-2.

$$\begin{array}{l} 89 = 1\,\bar{1}\,\bar{1} \\ 99 = 1\,0\,\bar{1} \\ \hline 1\,/\,\bar{1}/\bar{2}/1/1 \end{array} = 1\bar{1}\bar{2}11 = 8811$$

It is interesting to observe that there is absolutely no carry over in the entire operation.

(vi) After converting these numbers in the vinculum form, the multiplication using Urdhva sutra can be done with equal ease and simplicity even from left to right.

$$\begin{array}{r} 1\,\bar{1}\,\bar{1} \\ 1\,0\,\bar{1} \\ \hline 1/\bar{1}/\bar{2}/1/1 \end{array} = 1\bar{1}\bar{2}11 = 8811$$

(vii) The Nikhilam method can also be employed after converting the numbers in their vinculum form. (Ref. item 11).

$$\begin{array}{r} 1\,\bar{1}\,\bar{1} + \bar{1}\,\bar{1} \\ 1\,0\,\bar{1} + 0\,\bar{1} \\ \hline 1\,\bar{1}\,\bar{2}\,/\,1\,1 \end{array} = 1\,\bar{1}\,\bar{2}\,11$$

$$8811$$

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DISCUSSION : Vedic system provides large number of choices in almost every thing we want to do. The very first step is to recognize the pattern and pick up the most efficient method. This task of pattern recognition alongwith mental working activates the right half of the brain which has the capability of intution. Thus, with the regular practice of Vedic Maths and its most natural mental procedures, the hol istic development of human brain automatically takes place. Within a couple of years, it shall naturally develop the intutive faculty the most powerful facet of the human personality, which unfortunately remains undeveloped in most of the cases.

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18. MULTIPLICATION-THREE OR MORE NUMBERS

INTRODUCTION

The superfast Vedic procedure of multiplication can be further extended to multiply three, four or even more numbers simultaneously. Of course, the procedure becomes just a bit longer for larger numbers. This is another very special and unique feature in the cap of the (निखिलं नवतश्चरमं दशतः) 'Nikhilam Navatas caramam Das atah' Vedic Sutra. The following example shall serve the purpose of clearly demonstrating the very simple, straight forward step by step procedure. (Refer 3 & 9)

Illustration 40 :

Let us take a simple example of multiplying 98x97x96.

Step 1 : We write these numbers in the Vedic base system style. We get the deficiency of each number from the base by invoking the 'Nikhilam sutra'. Therefore, we get

(The base is 100)

98 - 02
97 - 03
96 - 04

/ /

Step 2 : For three numbers the product shall have three parts. Therefore two inclined lines (//) are drawn to demarcate the three parts of the answer.

Step 3 : We vertically multiply the three deficit numbers and the resulting number is the right hand portion of the answer. In this case (-02)x(-03)x(-04) = -24 is the required right hand portion. This is written down in the right hand space.

98 - 02
97 - 03
96 - 04

/ / $\bar{2}\bar{4}$

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Step 4 : The middle portion of the answer is obtained as follows. We multiply two deficit numbers at a time by rotation. The summation of these three small products gives the middle portion of our answer and is written within the two slashes.

In the present example, we get

$(-02 \times -03)+(-03 \times -04)+(-04 \times -02) = 6+12+8 = 26$

98 - 02
97 - 03
96 - 04

/26 /$\bar{2}\bar{4}$

Step 5 : The left hand portion of the resulting product is obtained by the algebraic addition of the first number with the deficiency of the other two numbers.

In this example the L.H.S. is $98+(-03)+(-04) = 91$.

The resulting number is written at the appropriate space

98 - 02
97 - 03
96 - 04

91/26/$\bar{2}\bar{4}$

Step 6 : Therefore, the answer obtained in one line by the simple mental arithmetical process is $912\bar{6}\bar{2}\bar{4}$.

Since the result may frequently have the Vinculum portion, the normalisation may be done in the usual manner to get at the final total positive answer. It is worthwhile to mention again that the 'Nikhilam sutra' is once again employed locally for normalising the Vinculum portion of any number.

Therefore $912\bar{6}\bar{2}\bar{4} = 912576$

OBSERVATION

An important point to be observed, though already emphasized in the previous chapter on Nikhilam sutra, is that in each of the steps number 3 and 4 respectively, the answer obtained had only two digit number. Since the base in this example is 100, which has two zeros and as such two digit answers of step 3 & 4 create no problem.

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Illustration : Multiply 988x996x995

988 - 012
996 - 004
995 - 005

979/128/240

= 979127 760

R.H.S. = (-012)x(-004)x(-005) = 240

Middle = (-012 x -004)+(-004 x -005)+(-005 x -012) = 128

L.H.S. = 988 -004 -005 = 979

19. DIVISION-NIKHILAM METHOD (Sutra 2) (Ref. 1,3,4 & 9)

Illustration 41 :

- Divide 216 by 89

89 / 216

Step 1 : Using Nikhilam we modify the divisor

Here MD = 11

Step 2 : We write as 89 / 216

MD 11

Step 3 : We draw a slash after 2 digits, and number the digits of the Dividend from R.H.S. and digits of Divisor are numbered from L.H.S.

3rd 2nd 1st
89 / 2 / 1 6

11
1st 2nd

Step 4 : Multiply the last digit of Dividend with the first and second digits of MD respectively and place the product below the previous digits (2nd and Ist column respectively) as follows

89 / 2/1 6
11 2 2
2/

Step 5 : On adding the digits of 2nd and Ist column we get result.

89 / 2/1 6
11 2 2
2/3 8

So Quotient = 2,
Remainder = 38

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Illustration 42 :

Divide 112 by 87

87 / 1/12
13 13
1/25

Hence Result

Q = 1
R = 25

Illustration 43 :

Divide 1011638 by 987

987 / 1011638

Step 1 : Using Nikhilam Sutra first we modify the given divisor (and get 013).

987 / 1011638
013

Step 2 : Here we have 3-digit divisor so we put a oblique line just after 3-digits from the R.H.S. of D

987 / 1011/638
013

987 / 1011/638
013 013
000
026
0412
1024/813 20

we note that the number of rows of the product is equal to the number of digits of D appearing after the oblique line drawn

Remainder is obtained on adding the carries i.e. 950

830
12
950

and our Quotient is Q = 1024
R = 950

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20. RECURRING DECIMALS (Sutra 1) (Ref. 1,2,4 & 9)

The Operator : The operator key number for computing the value of the recurring decimal can be obtained by applying the simple 'Ekadhikene Sutra'. The simple meaning of the Sutra 'Ekadhikene Purvena' is 'by one more than the previous one'. Thus for $\frac{1}{49}$ the operator key is one more than the previous digit (4). Therefore operator is 4+1 = 5 similarly for $\frac{1}{39}$ the operator = 3+1 = 4. For $\frac{1}{29}$, operator = 2+1 = 3. For $\frac{1}{19}$ operator = 1+1 = 2 etc.

THE PROCEDURE

'Ekadhikena Sutra' tells us that 'By one more than the previous one'. The first word' by is used both for multiplying and dividing. It is interesting to learn that this operator key can be used in both ways to compute the value of the recurring decimal. Here, we shall learn the method of using the operator key as a constant uniform multiplier.

Illustration 44 : Let us take the fraction $\frac{1}{39}$. In the result the last right hand most digit shall be 1 as the RHS digit is 9 and product of this RHS digit with the RHS digit of result is equal to 9, for recurring numbers. Further, the operator key is 3+1 = 4. The subsequent, step by step procedure is outlined below.

Step 1 : Put 1 as the extreme right digit and put a dot over it to indicate that it is the end of the set of recurring digits.

Step 2 : We multiply this digit by the operator and write down the results towards the left hand of the previous digit. If the resulting number has more than one digit we put down the extra digit below the next level. In this case 1x4 = 4 is written on the L.H.S. of 1 (the previous digit).

Step 3 : We multiply the previous digit (4) by the common constant operator key number (here 4). Therefore 4x4 = 16. We write down 6 on the LHS of previous digit 4 and excess digit 1 in the following manner (below the next level).

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Step 4 : We multiply the previous digit ($_1$6 4 1) by the operator 6x4 = 24 and add carry over digit 1 to yield the result 24+1 = 25. Once again we write 5 on LHS of the previous digit, 2 below the next level.

5 6 4 1
2 1

Step 5 : We multiply the previous digit (5) by the operator, 5x4 = 20 and add carry over digit 2 to the result 20+2 = 22. The result is written as before.

2 5 6 4 1
2 2 1 0

Step 6 : We multiply the previous digit 2 by the operator (4) therefore 2x4 = 8 and add the carry over digit 2 to 8, so 2+8 = 10. So our result wil become

0 2 5 6 4 1
1 2 2 1

Step 7 : We multiply 0x4 = 0, and the carry out digit 1 so that 0+1 = 1 etc. So we find that we have started getting the same digits after 0. Therefore we can put a dot over 0 to indicate that the set of recurring decimals starts from 0. The decimal is placed before 0. Thus the set of recurring decimal is -

$$\frac{1}{39} = 0.\dot{0}25641\dot{}$$

Duscission : Since 39 is a prime number, the behaviour of $\frac{1}{39}$. is different from other recurring numbers.

21. DIVISION - By Paravartaya Method (Ref. 1,3,4 & 9)

We have seen how Nikhilam Method can be applied for division by real big digits but however this is not at all suitable for division by smaller number. For division by smaller number - an equally easy and superfast vedic method is available. This

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is based upon the PARAVARTAYA SUTRA. This is the fourth Sutra in the total list of sixteen and means 'transpose and apply'. In other words we change the sign of the divisor digits and then apply the paravartaya method.

Procedure For Division Having Unity As Their First Digit : The procedure followed in such cases is exactly the same as we have seen in the case of division by Nihkilam method. The difference is only in initial steps, corresponding to the modification of division digits (MD). The stepwise process is illustrated below.

Illustration 45 : Divide 1235 by 112 $\quad 112\overline{)1235}$

Step 1 : We shall tabulate the divisor and divided digits as follows

(divisor digits) 112 $\overline{)}$ 1235 (divided digits 4-numbers)
3-numbers

Step 2 : (i) As stated earlier, the divisor is such that its first digit is one, while modifying the divisor ignore this digit. Thus modified divisor (MD) will contain only two digits (here they are 1 2).

(ii) Another modification in divisor is that all the remaining digits are taken down as digits of MD, but with their sign reversed in every case. Thus here MD is $\bar{1}\ \bar{2}$. In other words we transpose and apply as per Paravartaya Sutra.

(iii) Now MD contains two digits only, as such the oblique line shall be placed after two columns of dividend. As

	Columns						
3rd	2nd	1st		4th	3rd	2nd	1st
1	1	2	/	1	2	3	5

(M.D.) $\bar{1}\ \bar{2}$

Step 3 : All the remaining steps are the same as those already explained in the previous section (6). of division by Nikhilam. Namely (i) complete the row of products obtained

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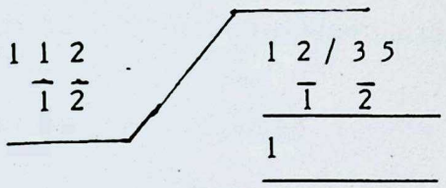
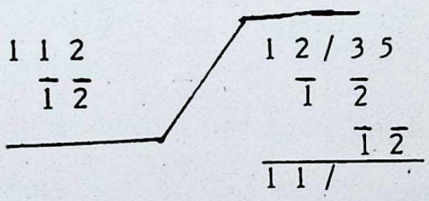
by multiplying with the first multiplier (appearing in the 4th column of D is directly our multiplier) by 2nd and 1st digit of MD and placing these below 3rd and 2nd column of D respectively.



(ii) The second row will be obtained by finding an operator multiplier (by summing the digits appearing the previous column (here 3rd) and multiplying the same with 2nd and 1st digits of MD, and placing these below 2nd and 1st column of D. Here operator is $1(2 + \bar{1} = 2 - 1 = 1)$

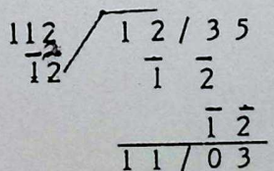


Since we have reached the first digit of D, all the operations of multiplications have been exhausted.

Note : This time since digits of MD are with negative sign the product may also have negative sign. Hence proper care of sign of product must be taken in handling the vinculum numbers.

(iii) Draw oblique line just below the oblique line of D and algebraically sum the digits, column wise, appearing on both sides of the line. The summation on R.H.S. of oblique line will be Remainder R and one on L.H.S. will be quotient Q.

Here is



Result

R = 03

Q = 11

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Procedure For Other Divisors : The previous examples had first digit as a unity in the divisor. But if this condition is not satisfied, we just need a small additional step as explained in the following example. Just like the multiplication, we use the Anurupena Sub-sutra in this situation.

Illustration 46 : Divided 1 3 5 4 by 1 8 4

Here 184 can be written as $2\bar{2}4$. In order to apply Paravartaya Sutra (first digit must be unity). For this we divide this divisor by 2. Thus the new divisor becomes $1\bar{1}2$.

```
 1 8 4          1 3 / 5 4
2/2 2̄ 4           1  2̄
 1 1̄ 2             4 8̄
   1 2̄
-------------------------
2/1 4/7 4̄   =  7 / 7 4̄  =  7 / 6 6
```

Here since divisor has been divided by 2, as such the raw quotient is also divided by 2 in order to get the final quotient. However, nothing is done to remainder, which remains unchanged. In this process we have again used the same upsutra 'Anurupena' which was used in the sub-base cases (Item 4.1) of multiplication.

Illustration 47 : Divided 16542305 by 9889

```
1 6 5 4 2 3 0 5   =  2 4̄ 5 4 2 3 0 5
9 8 8 9           =  1 0 1̄ 1̄ 1̄
1 0 1̄ 1̄ 1̄          2 4̄ 5 4 / 2 3 0 5
  0 1 1 1            0 2 2 /  2
                       0 4̄ /  4 4̄
                         0 /  7 7 7
                           /  0 2 2 2
                  -----------------------
                  2 4̄ 7 2 /  7 8 9 7      Q = 1672
                  1 6 7 2 /  7 8 9 7      R = 7897
```

DISCUSSION

By judicious use of vinculum, submultiplier or subdivisors as the case may be, we find through examples given above, that all the problems of divisions can be easily

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performed using Paravartaya sutra. Hence it is versatile sutra for performing division.

22. Algebraic Division (Ref. 1,3,4 & 9)

Introduction : In this method of division we use the same Vedic formulae Paravartaya Yojayet, which means,'transpose and apply'. It is interesting to observe that the Vedic method of division for algebraic equations is exactly similar to that for the numerical division as already explained in the previous section. As a matter of fact in Vedic method it seems difficult to draw a line where arithmetic stops and algebra starts. The following examples shall bring out the points of similarity and differences.

Illustration 48 : Divide $4x^2 + 15x - 13$ by $x + 1$

$x+1$) $4x^2 + 15x / -13$

-1

-4

-11

$4 + 11 / -24$

$Q = 4x + 11, \quad R = -24$

Illustration 49 : Divide $x^4 - x^3 + x^2 - 3x + 4$ by $x^2 - x + 1$. Here we operate the two coefficients at each stage of division.

x^2-x+1) $x^4 - x^3 + x^2 / -3x + 4$

$1 \quad -1$ $\qquad 1 \quad -1$

$0 \quad 0$

$0 \quad 0$

$x^2 + 0x + 0 / -3x + 4$

$Q = x^2, \quad R = -3x + 4$

Notes : Since the divisor has the x^2 term we can therefore have the remainder with x terms, as such we draw the oblique line, demarcating the remainder portion before the x term of the dividend.

2. Please note that we do not modify the power of x in the remainder terms.

3. To avoid error we also write even the row consisting of all zero terms.

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DISCUSSIONS

(1) The above examplesexplain the Superfast and easy vedic methods of division by using the Paravartaya sutra.

(2) The Vedic methods are rather easy and the chances of error in working with them are drastically reduced.

(3) It is interesting to point out that the so called Remainder theorem, which is specially taught in the algebra gives only the remainder of algebraic expressions. However the Paravartaya sutra goes much ahead and yields both the quotient and the remainder too.

(4) The Vedic methods go further ahead in providing us with the unique simple formula for checking the correctness of the results of our division both for arithmetic and algebraic problems.

(5) However the methods discussed so far are suitable for special cases and the general DHWAJANKA-METHOD is applicable for all cases without exception.

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23. SQUARING - DWANDWA YOGA

We can use the Urdhava method for squaring. There is another method for squaring using Dwandwa yoga sutra (or the Duplex combination).

To find the Square of the numbers we find the duplexes of the digits.

PROCEDURE :

Level 1 : If we have a one digit number. To Duplex (D) of one digit number we simply square the digit. For example 3, $D = 3^2 = 9$.

Level 2 : For two digit number. We find the duplexes of the digits as double the product of two digits, for ex.

$3\ 5,\ D = 2\ (3 \times 5) = 2\ (1\ 5) = 30$

Level 3 : For three digit number we find the duplexes as the sum of the square of the middle digit and double the product of the first and third digits as

$3\ 2\ 7,\ D = 2 \times 3 \times 7 + 2^2$

$= 4\ 2 + 4 = 4\ 6$

Level 4 : For four digit number we find the duplexes as the sum of,

double the product of the first and fourth digits, and,

double the product of the second and third digits as

$1\ 4\ 5\ 6 = 2 \times 1 \times 6 + 2 \times 4 \times 5$

$= 1\ 2 + 4\ 0 = 5\ 2$

Similarly we can extend it even for the larger digit numbers.

Illustration 50 : Calculate 931^2.

Step 1 : Duplex of I^{st} digit is

$1^2 = 1$

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Step 2 : Duplex of I^{st} and 2^{nd} digits i.e. double the product of two digits

We get for 3 1 ; $2 \times 3 \times 1 = 6$

$\therefore\ 931^2 = \ /\ /\ /6/1$

Step 3 : Duplex of I^{st}, 2^{nd} and 3^{rd} digits is

$931 = 2 \times 9 \times 1 + 3^2 = 18 + 9 = 27$

$\therefore\ 931^2 = \ /\ /\ {}_{2}7/6/1$

Step 4 : Duplex of 2^{nd} and 3^{rd} digits; double the product of two digits (level 2).

We have $9\ 3 = 2 \times 9 \times 3 = 54$

$\therefore\ 931^2 = \ /\ {}_{5}4/\ {}_{2}7/6/1$

Step 5 : Duplex of 3^{rd} digit only (level 1)

We get $9^2 = 81$

$\therefore\ 9\ 3\ 1 = 81/\ {}_{5}4/\ {}_{2}7/6/1$

Carry over the excess digits = 8 6 6 7 6 1

Hence $931^2 = 866761$

Illustration 51 : $3298^2 = 330\bar{2}$

$= 9/{}_{1}8/9/\bar{1}\bar{2}/{}_{\bar{1}}\bar{2}/0/4$

$= 1088\bar{3}\ \bar{2}04 = \underline{1087604}$

Illustration 52 : $20289^2 = 203\bar{1}\bar{1}^2$

$= 4/0/{}_{1}2/\bar{4}/5/\bar{6}/\bar{5}/2/1 = 412\bar{4}5\bar{6}\bar{5}21$

$= 41164321$

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DISCUSSION

Once again we observe the emphasis on mental working. This approach when regularly practised leads to a more agile and alert mind. The results can be obtained by working either from L.H.S. or R.H.S. as convenient.

24. CUBING

Introduction

In the previous section we have learnt now to square the number easily in one line. Similarly we can do the cubing of the numbers using the sub sutra Anurupyena, which means 'proportionately'. The following sections illustrate the step by step procedure of computing the cube of numbers.

In the general case the first term is cube of the first part (say a^3) and each of the subsequent three terms is obtained by multiplying the first term by the second part (b) and dividing by the first part. That is the subsequent terms are the G.P. of the ratio of 2 digits. Then we double the 2^{nd} and 3^{rd} terms, add and carry.

Illustration 53: 23^3.

23^3 = 8 12 18 27

24 36

8 36 54 27

= $8\ 3^6\ 5^4\ 2^7$

= 12167

I^{st} term = $2^3 = 8$

2^{nd} term = $8 \times 3/2 = 12$

etc.

Illustration 54: 34^3

34^3 = 27 36 48 64

72 96

27 108 144 64

= $27\ 10^8\ 14^4\ 6^4$ = 39304

Cube of a three digit number can be obtained in two steps.

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Illustration 55 : Cube the number 89.

Step 1 : Convert the given number (89) into vinculum form using Nikhilam sutra i.e.

$8\ 9 = 1\ \bar{1}\ \bar{1}$ (Vinculum form)

Step 2 : Break the vinculum form of the number into two parts as we do in the cubing of any number i.e. $1/\ \bar{1}\ \bar{1}$

Step 3 : Now we follow the cubing procedure directly

$$89^3 = 1/\bar{1}\,\bar{1}^3 = 1\ \ \bar{1}\,\bar{1}\ \ 1\,2\,1\ \ \bar{1}\,\bar{3}\,\bar{3}\,\bar{1}$$
$$\bar{2}\,\bar{2}\ \ 2\,4\,2$$
$$1\ \bar{3}\,\bar{3}\ \ 3^{6}\,3\ \ \bar{1}\,\bar{3}^{3}\,1$$
$$= 1\,\bar{3}\,0\,5\,0\,\bar{3}\,\bar{1}$$
$$= 704969$$

We can see how convenient it is to handle the smaller digits.

Alternately

We can break the vinculum form of number as $1\ \bar{1}/\ \bar{1}$ and then use the same procedure, we get

$$1\ \bar{1}/\ \bar{1}^3 = 1\,\bar{3}\,3\,\bar{1}\ \ \bar{1}\,2\,\bar{1}\ \ 1\,\bar{1}\ \ \bar{1}$$
$$\bar{2}\,4\,\bar{2}\ \ 2\,\bar{2}$$
$$1\,\bar{3}\,3\,\bar{1}\ \ \bar{3}\,6\,\bar{3}\ \ 3\,\bar{3}\ .\ \bar{1}$$
$$= 1\,\bar{3}\,0\,5\,0\,\bar{3}\,\bar{1} = 704969$$

Notes : (1) The method of cubing can also be applied from R.H.S. to L.H.S.

(ii) We can directly use the same method by using the base.

(iii) Direct method of ratio for cubing can also be used.

(iv) Once again, as usual we find the 4 alternative methods available in the Vedic Maths approach, giving a choice to the user.

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25. STRAIGHT DIVISION

There is yet another method of division in Vedic system. This involves a bit more of mental working but the answer comes straight away in one line. The method is evolved by the combination of two Vedic sutras, namely Urdhava Tiryak and Dhwajanka (meaning on top of the flag).

Illustration 56 : Divide 4 6 9 5 6 by 6 5 .

6 5 ⟌ 4 6 9 5 6

Step 1 : Out of the divisor 65, we put down first digit (i.e. 6) in the divisor column as operator and other digit (i.e. 5) on top of the flag as flag digit as shown below:

Flag Digit
5 ⟌ 4 6 9 5 6

(operator) 6

The entire division is to be done by 6.

Step 2 : As one digit has been put on the top we allot one place (at the right end of D) to the remainder portion of the answer and mark off from the other digits by vertical dots

- Here last digit 6 has been marked off

5 : 4 6 9 5 : 6 :
6

Step 3 : (i) We divide 46 (of dividend) by 6 and get 7 as the quotient and 4 as remainder, we put down 7 as the first quotient digit and prefix the remainder (4) up before the next digit (i.e. 9) of the dividend. Then our actual gross dividend (G.D.) is 49 for next step.

6 5 : 4 6 9 5 : 6 :
4
7

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(ii) from 49 we subtract the product of the flag digit (i.e. 5) and the first quotient digit (i.e. 7). So we get 49 -

$$7 \times 5 = 35$$
$$14$$

Thus 14 is our net dividend for further operation.

Step 4 : (i) Now divide this net dividend i.e. 14 by 6 we get 2 as second quotient digit and 2 as remainder which is placed before the next digit of dividend (i.e. 5). Now our Gross dividend is 25.

$$6^{5} : 4\ 6\ 9\ 5 : 6 :$$
$$4\ 2$$
$$7\ 2$$

(ii) Again from 25 we subtract the product of the flag digit (5) and the 2nd quotient digit (i.e. 2). So we get $25 - (2 \times 5) = 25 - 10 = 15$. This is the net dividend.

Step 5 : (i) On dividing 15 by 6 we get 2 as third quotient digit and 3 as remainder to be placed before 6. Now 36 is our next gross dividend

$$6^{5} : 4\ 6\ 9\ 5 : 6 :$$
$$4\ 2\ 3$$
$$7\ 2\ 2$$

(ii) Out of 36 we subtract the product of the flag digit (5) and the 3rd quotient digit (i.e. 2). So we get $36 - (2 \times 5) = 36 - 10 = 26$.

Hence 26 is the remainder.

$$6^{5} : 4\ 6\ 9\ 5 : 6 :$$
$$4\ 2\ 3$$
$$7\ 2\ 2 : 2\ 6 :$$

Therefore Q = 7 2 2

R = 2 6

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Illustration 57 : Divide 5120 by 64.

$6^{4} : 5\ 1\ 2 : 0 :$

$\quad\quad 3$

$\quad\quad 8\ 0 : 0 :$

The second net dividend is 0 [32 - 8 x 4]

Illustration 58 : Divide 7648 by 94.

$9^{4} : 7\ 6\ 4 : 8 :$

$\quad\quad 4\ 3$

$\quad\quad 8\ 1 : 3\ 4 :$

Decimal form : All the three Vedic methods of division including the Vedic DHAWAJKAM METHOD can be easily extended to get the result in decimal form by continuing the process of division of the remainder. We place the additional zeros after the dividend. The following examples shall illustrate the process for Dhawajkam method.

Illustration 59 : Divide 8352 by 79 to three places of Decimals.

$7^{9} : 8\ 3\ 5 : 2 . 0\ 0$

$\quad\quad 4\ 10\ 83$

$\quad\quad 1\ 0\ 5 : 7\ 2\ 1$

The process of solving these questions is same. We put the decimal point as soon we start the division of the remainder.

Illustration 60 : Divide 711.023 by 38 to three places of decimals.

$3^{8} : 7\ 1\ 1 . 0\ 2\ 3$

$\quad\quad 49\ 6\ 11$

$\quad\quad 18 . 711$

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Illustration 61 : Divide 7.5 by 54 to 5 places of decimals.

4 : 7 . 5 0 0 0 0
5
2 6 8 8 8
. 138888

26. AUXILIARY FRACTIONS

In case of certain vulgar fractions we observe that Ekadhiken Purven Sutra helps us greatly to find their exact values. However, the fractions are of great varieties, and, a few more applications are presented.

Here we shall replace the denominator by its Ekadhika (i.e. drop the last digit and increase the penultimate one by 1) and make a consequential alteration in the division procedure.

Illustration 62 : For $\frac{1}{19}$ the Auxilliary fraction is $\frac{1}{2}$.

$\therefore \frac{1}{19} = .\dot{0}5263157894736842\dot{1}$

For $\frac{1}{29}$ the A.F. is $\frac{1}{3}$.

$\therefore \frac{1}{29} = .\dot{0}3448275862068965517241379\dot{3}$

For $\frac{37}{59}$, A.F. is $\frac{3.7}{6}$

For $\frac{3}{59}$, A.F. $= \frac{0.3}{6}$

For $\frac{56}{15999}$, A.F. $= \frac{.056}{16}$

For $\frac{21863}{49999}$, A.F. $= \frac{2.1863}{5}$

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To find the values of such factors we shall take the last example. Hence $F = \frac{21863}{49999}$ and A.F. $= \frac{2.1863}{.5}$. As we have dropped 4 nines from original denominator, we have to divide the numerator of the latter in bundles, so to say, of 4 digits each by 5; and whatever remainder there is, has to be prefixed not to any particular quotient-digit but to the bundle just already reached.

Thus, we take up 2.1863 to start with and divide it by 5, we get

Q = .4 3 7 2 and R = 3

So 0.4372 is the first quotient group and 3 as the remainder. We prefix this remainder to that group and say

3.4372 and then divide this dividend (namely 34372) by the same divisor 5; and we get

$3.4\ 3\ 7\ 2\ \ {}_2 6\ 8\ 7\ 4\ \ {}_4 5\ 3\ 7\ 4$

i.e. 6874 is our second quotient-group and 2 is the second remainder, which is therefore prefixed to the second quotient-group. And we continue this process with as many groups as we need. Thus we have

$3.4\ 3\ 7\ 2,_2\ 6.8\ 7\ 4,_4\ 5\ 3\ 7\ 4,_4\ 9\ 0\ 7\ 4$

and so on to any number, or tens, or hundreds or thousands etc. of decimal places.

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27. SQUARE ROOTING

To find the square root of the number using the Vedic Sutra is very simple, easy and interesting. The sub sutra named Dwandwa Yoga (द्वन्द्व योग) or Duplex process has two meanings. First one is by squaring and the second one is cross multiplication. This sutra is very easy to apply. Using this sutra we have seen how to find the squares of the numbers. (Item 10).

The step by step process of computing the square root is illustrated in the following examples.

Illustration 63 : Find the Square root of 4489.

Step 1 : First we make the pairs of two digits each. R.H.S. to L.H.S. from the given number, and write as 44 : 89 :

The first pair of two digits from L.H.S. left over or the single digit (if number of digits is odd) is written outside the dots and remaining pairs are written in between the dots.

Step 2 : Now we find the square of the first nine natural numbers as 1, 4, 9, 16, 25, 36, 49, 64, 81 and choose one of them which is closest and lesser than the first pair

(i) In this case square of 6 is suitable. So 6 is the first quotient digit. Which is placed with in the pair of dots. After this step we find the remaining part of the square root using straight division process (Item No. 12). So subtract the square of 6 (i.e. 36) from 44 we get (44 - 36 = 8) 8 as a remainder which is prefixed as the first digit of the next pair (i.e. 8). Hence 88 is the dividend.

(ii) For finding the divisor we double the first quotient digit (i.e. 2 x 6 = 12) obtained in step 2 (i) and write as follows

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$$\begin{array}{l} 44 : {}_{8}89 : \\ 12 \\ \hline : 6 : \\ \hline \end{array}$$

So 12 is our divisor. The entire division is done by 12.

Step 3 : Now divide 88 by 12 we get 7 as a quotient and 4 as the remainder which is prefixed to the next digit 9. So 49 is our next gross dividend

$$\begin{array}{l} 44 : {}_{8}{}^{8}\ {}_{4}{}^{9} : \\ 12 : \\ \hline : 6 : 7 : \\ \hline \end{array}$$

Now subtract the square of the quotient digit (i.e. $7^2 = 49$) obtained in (Step 3(1)) from the gross dividend 49, we get (49 - 49 = 0) 0 as quotient and 0 as remainder. Hence the square root is 67. Which is an exact square.

Notes : (1) We have just seen how easy it is to find the square root of any big number.

(2) If a number contains n digits

(i) If n is odd than the square root of a number shall have $\frac{n+1}{2}$ number of digits.

(ii) If n is even than the square root of number shall have $\frac{n}{2}$ digits.

(iii0 If the number is of <u>pure decimal</u> then the number of digits in the square is double that in the square root.

(iv) If the square root is not an exact number, the decimal shall be put after the number of digits equal to the number of pairs formed.

Illustration 64 : Find the square root of 17.

$$\begin{array}{l} 17 : 0\ 0.0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 8 : \quad : 1\ 2\ 3\ 2\ 2\ 6\ 7\ 3 \\ \hline : 4 \quad : 1\ 2\ 3\ 1\ 0\ 5\ 6 \end{array}$$

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the net dividendsat each stage are

$$10$$
$$20 - 1^2 = 20 - 1 = 19$$
$$30 - 2 \times (1 \times 2) = 30 - 4 = 26$$
$$20 - (2 \times (3 \times 1) + 2^2) = 20 - 10 = 10$$
$$20 - (2 \times (1 \times 1) + 2 \times (2 \times 3)) = 20 - 14 = 6$$

and so on. Hence the square root of 17 is 4. 1231056 which is not an exact square root.

Illustration 65 : Find the square root of 16641.

```
 1 : 6 6 4 1 ;
2  : 0 2 4 8
-------------
 : 1 : 2 9.00 :   (Complete)
```

The result is 129.

Notes : (i) After making the two digit pairs of the number from R.H.S. to L.H.S. if a single digit is left then it is counted as a pair (of single digit).

(ii) When we divide 6 by 2 after step 2, we write 2 as a quotient digit instead of 3, if we write 3 as Q we get remainder 9 and the next gross dividend is 06 which is smaller than the square of the Q digit 3 (i.e. 6-9) which would give the negative dividend value. So we take 2 as the quotient digit

(iii) Due to the same reason we do not divide the net dividend 8 by 2 (obtained in steps 5(ii)).

Vinculum Approach : We can also proceed by using vinculum numbers.

```
1 : 6 6  4 1 :
    0 0 1̄  0
2
-------------
1 : 3 1̄ 0 0
```

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Square root = $13\bar{1}$ = 129 ; is an exact square root

Net dividend

$1^2 = 1$

$2 \times 1 = 2$ Divisor

$06 - 3 \times 3 = \bar{3}$

$\bar{1}\,4 - 2 \times 3 \times \bar{1} = -6 + 6 = 0$

$01 - (2 \times 3 \times 0 + \bar{1} \times \bar{1}) = 1 - 1 = 0$

Illustration 66 : Find the square root of the number .000015.

First we make the pairs of the number as .00 00 15 gives 15 as the first non zero digit pair. The square root of the two pair of zero is .00 according to the discussion. So we find only the square root of 15.

Hence

```
    : 1 5 : 0 0 0 0
6 :     6   12 14 10
  : 3   : 8 7 3 0 --
```

Hence the square root is .0038730 of the given number.

Vinculum Approach : Let us try to solve the Illustration 52 again by using vinculum approach.

```
. 00  00  15  for 15
        15; 0  0_0  0_0
            6  6  3  6  3
      6
-------------------------
        3. 9 3 3 0 2
```

= 3.8730 etc.

Hence .0038730 is the square root.

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28. SIMPLE EQUATIONS

Using the simple Vedic sutras it is possible to solve a large class of equations directly through mental computations. Several types have been covered by Swami Ji in his book. (Ref. 4). Just a couple of examples are presented here.

Illustration 67 : $\frac{5x + 3}{2x + 7} = \frac{4x + 9}{7x + 5}$

Since $(5x + 3) + (4x + 9) = (2x + 7) + (7x + 5)$

$= 9x + 12,$

we can at once say, using Shunyam sutra,

$\therefore \underline{9x + 12 = 0}$

whence $x = -12/9 = -4/3$

But we are dealing with a quadratic : its second solution can be obtained as follows. Write the equation as

$$\frac{N_1}{D_1} = \frac{N_2}{D_2}$$

Now $N_1 \sim D_1 = (5x + 3) - (2x + 7) = 3x - 4$

and $N_2 \sim D_2 = (7x + 5) - (4x + 9) = 3x - 4$

(Where $\sim$ means that we subtract the smaller term from the larger and for this purpose we can give the coefficient of x a greater weight than the constant term.)

$\therefore$ the second solution is

$$\underline{3x - 4 = 0}$$

2) $\frac{5x - 1}{2x + 7} = \frac{8x + 9}{11x + 1}$

$\therefore \quad 13x + 8 = 0$

i.e. $\underline{x = -8/13}$

and $3x - 8 = 0$;

$\underline{x = 8/3}$

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29. **FACTORISATION** : Factorisation of quadratics. Two sutras are needed.

आनुरूप्येण (anurupyena) i.e. proportionately, and

आद्यमाद्येनान्त्यमन्त्येन (adyamadyenantyamantyena), i.e. the first by the first and the last by the last. (Ref. 4). E.g. to factorise $2x^2 + 11x + 12$, we note that there are three coefficients, 2, 11, 12. Split the middle of these into two parts in such a way that the ratio of the first coefficient to the first part is the same as the ratio of the second part to the third coefficient. i.e. $2x^2 + 11x + 12 = 2x^2 + 3x + 8x + 12$, where $2:3 = 8:12$.

Then this ratio, 2:3, gives us one of the factors, $2x + 3$.

The second factor is obtained by:

(i) dividing the first coefficient of the quadratic (2) by the first coefficient of the factor (2). i.e. $2/2 = 1$, and this is the coefficient of x, and

(ii) dividing the last coefficient in the quadratic (12) by the last coefficient in the factor (3), i.e. 12/3 gives 4, whence the second factor is $x + 4$

$\therefore 2x^2 + 11x + 12 = (2x + 3)(x + 4)$

Illustration 68:

1) $3x^2 + 19x + 20$

We have $3:4 = 15 : 20$. Hence $(3x + 4)$ is a factor.

and $3/3 = 1$; $20/4 = 5$, whence $3x^2 + 19x + 20 = (3x + 4)(x + 5)$.

2) $E = 2x^2 + 9x + 9$

Since $2:6 = 3 : 9$,

$E = (x + 3)(2x + 3)$

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Further the solution of transcendental equations and partial differential equations have also been obtained (Ref. 8).

30. VEDIC NUMERICAL CODE

It is a matter of interest to note that Vedic scholar of ancient time did not use figures (for big numbers) in their numerical notations but preferred to use Sanskrit alphabet to represent various numbers. This was perhaps done to facilitate the mathematical knowledge stored and transferred on to subsequent generations by way of language, understood to mankind. Thus preservation of their knowledge was better managed and understood by subsequent followers. Sutras and Sanskrit verse are in fact treasures of such kind which can be easily decoded and assimilated even to the modern world.

In this context, it is interesting to point out the honour being given to the Sanskrit language by the American Association for Artificial Intelligence. The full cover page of the AI magazine (volume 6 No. 1 spring 85), an official publication of the American Association for the artificial intelligence is decorated with Sanskrit shlokas. This particular issue also contains a research article of 10 pages by Dr. Rick Briggs, who is a computer scientist at NASA, American Space Research Centre, CALIFORNIA, on, "Knowledge representation in Sanskrit and Artificial Intelligence "

THE CODE

The code system in Hindi could be written down as :

क	ट	प	-1	य
ख	ठ	फ	2	र
ग	ड	ब	3	ल
घ	ढ	भ	4	व
ङ	ण	म	5	श
च	त		6	ष
छ	थ		7	स
ज	द		8	ह
झ	ध		9	
		क्ष	0	

Note : Further न and ञ are also be coded as 5

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The whole code is contained in verse

कादिनव टादिनव पादिपंचक याद्यष्टक क्ष शून्यम् च।

This code system can also be written in English as follows :

(i) ka, tta, pa and ya all denotes 1,
(ii) kha, dah, pha, ra all denotes 2,
(iii) ga, da, ba, la all denotes 3,
(iv) gha, dha, bha, va stands 4,
(v) gna, na, ma and sha denotes 5,
(vi) ca, ta and ksha denotes 6,
(vii) cha, tha, sa denotes 7,
(viii) ja, da and ha stands 8,
(ix) jha, dah denotes 9,
(x) Ksa (or Ksudra or shunyam) for 0

The vowels are ignored and in case of conjunct consonants the last consonant is alone counted.

Illustration 69 : Thus pa pa is 11, ma ma is 55, ta ta is 11, ma ry is 52 and so on.

It was left to the author to select proper consonant or vowel. This gave lattitude to the writers to frame the language of their writings to acquire more than one meanings. For instance, kapa, tapa, papa and yapa all mean 11.

Applications : We now take three illustrations which when decoded provide interesting mathematical insight :

(i) केवलैः सप्तकं गुण्यात्।

Here Saptakam means 7 and Kevalaih represents 143. We are told that in case of 7 out multiplicand should be 143.

Applying Ekanyana corollary of Nikhilam all this means as for 7 (as denominator) multiply 143 by 999 (Refer item 5). This is the answer for -

$$\frac{1}{7} = .\dot{1}\ 4\ 2\ 8\ 5\ \dot{7}$$

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(ii) कंसे क्षामदाहखलैर्मलैः

Here Kanse means 17 ksama-daha-khalairmalaih means 0 5 8 8 2 3 5 3.

Thus $\frac{1}{17}$ is obtained by 0 5 8 8 2 3 5 3 x 9 9 9 9 9 9 9 9

i.e. $\frac{1}{17}$ = . 0 5 8 8 2 3 5 2 9 4 1 1 7 6 4 7

(iii) Lastly a Shloka for obtaining value of $\pi/10$.

गोपीभाग्यम् मधुव्रात शृङ्गिशोदधि सन्धिग
खल जीवित खाताव गलहाला रंसधर ॥

This Shloka has two sets of literal meaning, first in praise of Lord Krishna and second for the Lord Shiva. Third meaning (after decoding) gives value of $\pi/10$ to 32 places of decimals as below -

$\frac{\pi}{10}$ = . 31415926535897932384626433832792 -------

Further, research is revealing very interesting results. For instance the Sacred Gaytri Mantra, when decoded yields the sacred 108 number. Further details are presented in Ref. 10.

Observations : Dr. V.P. Dalal (of Heidelberg University Germany) after knowing above Shloka's merit commented as follows -

It shows how deeply the ancient Indian Mathematicians penetrated, in the subtlety of their calculations, even when the Greeks had no numerals above 1000 and their multiplications were so very complex, which they performed with the help of the counting frame by adding so many times the multiplier; 7x5 could be done by adding 7 on the counting frame 5 times. etc.

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31. EVALUATION OF DETERMINANTS : (Ref. 1,8 & 10)

Illustration 70 : Evaluate the following determinant.

$$E = \begin{vmatrix} -26 & -11 & 17 \\ -3 & 7 & 1 \\ 2 & 4 & 3 \\ 5 & 8 & -4 \end{vmatrix}$$

The cofactors of the first two rows are computed by using Urdh va Sutra and written at the top.

$-3 \times 4 - 2 \times 7 = -26$ etc.

Further, the vertically and crosswise sutra is used and the cofactors (written in the top row) are multiplied by the third row.

$$E = (-26) \times (-4) + (-11) \times 8 + 17 \times 5$$

$$= 277$$

The Urdh va method can be extended to evaluate the determinants of size 4×4 and 6×6 also. The higher size determinants can be computed by first reducing their size using the pivoting method.

Illustration 71 : Similarly we can evaluate the following determinant.

$$\begin{array}{cccc} & + & - & + \\ & -16 & -17 & -28 \\ + & -3 & -32 & +12 \\ \hline 2 & 5 & 7 & 8 \\ 3 & 6 & 2 & 4 \\ \hline 3 & 5 & 7 & 6 \\ 1 & 6 & 9 & 7 \\ \hline 13 & 3 & -5 & \\ 20 & -1 & & \\ 15 & & & \end{array}$$

And D = 186

The entire effort and time required in writing again and again is totally eliminated and the computations are mentally performed to straight arrive at the result.

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32. SIMULTANEOUS LINEAR EQUATIONS : (Ref. 1,4,8 & 10)

For 3 equations in 3 unknowns we can use the partitioning technique, and compute the 6 cofactors of two portions (written outside) using Urdh va Sutra (see Sec. 17). Further, the four 3 x 3 determinants are computed mentally by using Urdh va, expanding by using the appropriate column. In the third step the values of the unknown variables are obtained directly. The whole working is shown below:

Illustration 72:

	xD	yD	zD		D	
-8	4x	-2y	+3z	=	8	-5
14	2x	-3y	+ z	=	1	-17
13	3x	+2y	- z	=	3	4

zD = 8 x 13 - 14 x 1 - 8 x 3 = 66 etc.

$\therefore$ D = 33 ; xD = 33 ; - yD = -33 ; zD = 66

Whence x = y = 1 ; z = 2

This method can be further extended to solve for four unknowns. Further, using the pivoting technique (Ref. 8) it is now possible to solve with equal ease and simplicity even eight or ten simultaneous equations. The total working space required is hardly two pages (Ref. 10).

33. INVERSION OF MATRICES : (Ref. 8 & 10)

The Urdh va Sutra can be used to compute the inverse of a 3 x 3 matrix following the well known procedure of writing the inverse of a 2 x 2 matrix. Through mental working process the cofactors are computed and divided by the determinant of A.

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Illustration 73:

$$A = \begin{matrix} 1 \\ 3 \\ 1 \end{matrix} \begin{matrix} 5 \\ -13 \\ 8 \end{matrix} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 5 & 1 & 4 \\ 3 & 1 & 0 & 2 \\ 1 & 3 & 1 & 4 \end{bmatrix} \begin{matrix} 5 \\ 2 \\ \\ -2 \end{matrix} \begin{matrix} \\ 4 \\ 0 \\ \end{matrix} \quad 5$$

$$\therefore A^{-1} = \frac{-1}{33} \begin{bmatrix} -4 & -7 & -10 & 15 \\ 2 & -13 & 5 & 9 \\ -22 & -22 & 11 & 33 \\ 5 & 17 & -4 & -22 \end{bmatrix}$$

Here cofactor are evaluated using the pyramidal method of evaluating determinants.

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OPINIONS OF THE PARTICIPANTS

Everybody who has learnt a little about the Vedic Mathematics has been thrilled by the superfast, almost magic speed calculation procedures of the Vedic Mathematics.

The Spiritual Study Group at Roorkee has been organising, Elementary and Advanced Course on Vedic Mathematics since May, 85. All the participants are asked to fill up the response forms comparing the present system of Mathematics with the Vedic system. A total of 11 questions were asked. The minimum score in favour of Vedic Mathematics for any question in any course so far was 80%. Most of the question got 100% favour. Further, comments of participants are motivating and inspiring for spreading Vedic Mathematics and its systematic teaching in schools and colleges.

A few glimpses of the participants opinions :-

- A boon for all competitive examinations.
- Could never imagine it would be so interesting.
- Very pleasant to have this course at the university.
- How advanced our ancestors were even in Mathematics.
- If Vedic Maths was taught in every School, computers would have developed in India.
- Excellent course.
- Want to learn advanced topics.
- Increases devotion towards our INDIA and its ancient culture.
- Now I wish to learn Sanskrit to know more about our ancient scientific developments.
- Every person on earth should have the knowledge of Vedic Mathematics.
- It was a boon to learn Vedic Mathematics in Dushera holidays (I had planned for my regular studies).
- Want to learn more about Vedic Mathematics like factorisation etc., which are difficult in modern system.

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- Amazing speed, very wide applications covering almost all the mathematical operations.
- A dynamic step for its introduction in educational system is a must.
- Has created an interest to know more about it.
- It can be of great use for children if made part of syllabus.
- Mysterious nature of numbers. After little practice problems can be mentally solved at magic speed.
- Very educative, has much scope for coming generation to start its practice from beginning.
- It is a strong proof of our rich cultural heritage.
- By Vedic methods maths which is a burden, be-comes a joy for the children.
- This is the most interesting and thrilling time of my life. Being a mathematics teacher I have interest in maths but this Vedic Mathematics has thrilled me. When I came in contact with the Vedic Mathematics, it gave such a pleasure, which I can't explain in words.

A teacher of Adarsh Bal Niketan

- The lectures were eye openers and most thrilling.
- A glimpse of the lofty trekking in Himalayas of ancient knowledge in Maths, developed with spiritualism. Cudos to SSG.

A Professor of University

- I have enjoyed all aspects of Vedic Maths course - beautifully blissful two weeks.

Mrs. R. Leahy
Iowa U.S.A.

- A nery enjoyable course - looking forward to learning deeper applications.

S. Olvsson
Sweden

- I enjoying practicing Vedic Maths as much as I enjoy Meditating.

R. Kneedler

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DISCUSSIONS

The essence of Vedic system is that it is simple, direct, one line and mental. The Vedic sutras are applicable to a wide variety of problems. They are very easy to understand and a delight to use.

1. The property of numbers is very extensively exploited in the Vedic Mathematics particularly in the field of computations. The Vedic approach has many special methods - indeed many more of them than conventional methods. This large flexibility of methods finds itself reflected in the mind, when approaching problem from Vedic view point. This indeed is the great benefit of the approach. One wonders at the reason of the supreme simplicity and ease of the Vedic method, which is really lacking in most of our usual methods.

2. Vedic maths is perfectly adaptable in oral teaching and mental calculation. They of course demand regular practice of simple problems. They are perfectly in tune with the oral Vedic tradition suplemented by terse statements of important points-called Sutras, which act as an aid to the memory.

3. Even a little exposure to the Vedic maths approach coupled with some practice clearly shows us that we are dealing with a entirely new and direct way of thinking.

4. In May 1985 when the first Vedic Mathematics course was being organised at the University campus, one of our students from Nepal, Sh. Bodhswarup (of B.E. Final Year Civil) who had just heard about the Vedic Mathematics book by Shankaracharya Ji Maharaj knew about the Vedic general method of multiplication (based on the Urdhava sutra). He had a perfect mastery over this method. He also introduced to us the Sudha upsutra and its use for Vedic method of addition subtraction, and compound addition and subtraction. Mr. Bodh had learnt these Vedic methods from his spiritual Guru who also was not aware of the Shankaracharya Ji's publication.

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Further, Dr. Khanna of B.H.U. also informed us about his working knowledge of the general Vedic method of multiplication.

Dr. Ram Chandra from Jodhpur, who was not aware of the Shankaracharya Ji's publi cati on, explained to us the Vedic Method of checking the results of any multiplication. Later on we found the procedure to be the straight application of the Gunit Sutra (no. 15).

In Bhaskaracharyas Leelavati, one of the methods of multiplication uses the deviation of the number from the base, which is very close to the Nikhilam method of Vedic Maths.

The alpha numeric coding system is based on the verse called Katadi (कटादि) sutra in ancient "Phalit Jyotish" (फलित ज्योतिष) - Predictive astrology. There are several applications of the same. For example one of the matrices of (size 8x12) 96 elements giving the relative weightage, due to the inter planetry positions, has been codified in the simple four line Sanskrit verse. All the 96 elements of this matrix can be obtained at any time by simply decoding the verse.

Further, Dr. Rahul from Gorakhpur has recently decoded the Gayatri using the same key. The first part of the Shatakshara, Gayatri Mantra has 24 letters. The numerical value of thus adds up to the auspicious number 108. Many persons are using Vedic addition process with slight variations.

All these indicate clearly that the different facets of Vedic Mathematics have been scattered and shattered due to mass destruction and killings at the ancient Vedic seats of learning. There is a need to do further extensive research both in applications and cross linkage. But, whatever is available today is sufficient to make an excellent start both at the teaching as well as at research level.

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5. Vedic Mathematics, due to its multi choice facility, provides wonderful opportunities for the development of the innovative and research faculty of the young students. A large number of Vedic Maths participants have been coming forward with very interesting and useful links. The age and qualification is no barrier in this process. The extensions of the Nikhilam method for multiplying three or more numbers and the use of different bases and subbase have all been provided by one of our UG students.

It shall be interesting to point out the recent research work of Sh. Sanjay, who has just obtained his Engineering Diploma. Mr. Sanjay has very successfully extended, the Vedic method of cubing, to find any power of a given number even beyond the computer capability. Further, he has found that the Urdhava method of solving linear simultaneous equations can be used for solving for even 8 or 10 unknowns using the Pivoting technique. The whole working can be completed in just 2 pages.

6. The Vedic Maths is primarily for mental working and has the potential of developing the human computer to wonderful level where the results of the problems flow very naturally, with the least amount of effort, in the typical Vedic way. This approach provides a corrective methodology to the problem of mental slavery to the calculators.

Further, Vedic algorithms have considerable potentials for automatic computations and small computers can handle much bigger problems due to the ease & simplicity of processes and lesser memory requirements.

Let us learn, teach and preach Vedic Mathematics which children learn with smiles on their face and joy in their hearts.

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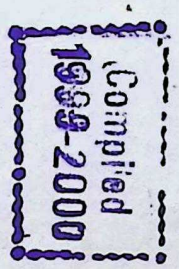

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